Pure Paper 1

Q1 Given that $\mathbf{p} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, show that $3\mathbf{p} - 2\mathbf{q}$ is parallel to $\mathbf{r}$.

[2 marks]

Q2 a) A student says that $\sqrt{x^2} = x$ for all x.
Prove by counter-example that the student's statement is incorrect.

[1 mark]

b) Prove that when n is a positive even number,
$2n^2 + 2n + 6$ is never exactly divisible by 4.

[3 marks]

Q3 Differentiate $f(x) = 4x^2 - 4x + 3$ from first principles.

[4 marks]

Q4 Clearly showing your working, find the exact solution(s) of the equation:

$$2\log_5(2x - 1) = 1 + \log_5(3 - x)$$

[4 marks]

Q5 $p(x) = 2x^3 + ax^2 + bx + 18$ is exactly divisible by $(x - 3)$ and $(x + 2)$, where a and b are integers.

a) By using the Factor Theorem, find the values of a and b.

[3 marks]

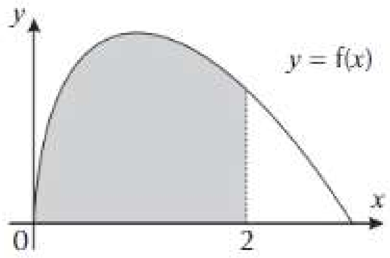
b) Hence, fully factorise $p(x)$.

[1 mark]

Q6 a) Find the integral $\int \frac{9x^2 - 3x^3}{\sqrt{x^3}}\,dx$.

[3 marks]

The diagram shows part of the graph of $f(x) = \frac{9x^2 - 3x^3}{\sqrt{x^3}}$.



b) Given that f(x) passes through the origin, find the exact value of the area of the shaded region bounded by the curve f(x), the x-axis and the line $x = 2$. Give your answer in the form $p\sqrt{q}$, where p and q are rational numbers.

[3 marks]

Q7 A, B and C are the vertices of a triangle.

The position vectors of A and B are $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}3\\7\end{pmatrix}$ respectively.

a) Find $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$, giving your answer as a simplified surd where appropriate.

[3 marks]

b) Given that $|\overrightarrow{AC}| = 2\sqrt{3}$ and $|\overrightarrow{BC}| = 4$, find the angle at vertex A in degrees to one decimal place.

[2 marks]

c) Hence, find the area of the triangle ABC to one decimal place.

[2 marks]

Q8 The straight line l intersects the curve $f(x) = 2 - 6x + 8x^2 - 2x^3$ at two distinct points, P and Q. Given that l is a tangent to f(x) at P and the coordinates of P are (1, 2):

a) find the equation of line l,

[4 marks]

b) find the coordinates of point Q.

[3 marks]

Q9 a) Find the binomial expansion of $(2x + 3)^5$.

[4 marks]

b) State the binomial expansion of $(2x - 3)^5$.

[1 mark]

c) Hence solve the equation

$$(2x + 3)^5 - (2x - 3)^5 = 475x^4 + 2138x^2 + 501$$

[3 marks]

Q10 Given that $f(x) = \frac{2x-4}{4-x}$

a) Show that $f(x) = a + \frac{b}{4-x}$,
where a and b are integers to be found, clearly showing your working.

[2 marks]

b) Sketch the graph of $y = f(x)$, clearly indicating on your diagram any points of intersection with the coordinate axes and any horizontal and vertical asymptotes.

[3 marks]

Q11 During an experiment, the pressure inside two separate tanks filled with gas varies over time.

The pressure in tank P is modelled by the equation $p(t) = 72 + 10t - 2t^2$ and the pressure in tank Q is modelled by $q(t) = a + bt$. The pressure in each tank is measured in kilopascals (kPa) and t is measured in minutes after the start of the experiment.

When $t = 0$, the pressure in tank Q is measured as 90 kPa.
When $t = 7$, the pressure in tank Q is measured as 34 kPa.

a) Determine the values of a and b.

[2 marks]

b) For how long during the experiment is the pressure in tank P greater than or equal to the pressure in tank Q?
Give your answer in minutes and seconds to the nearest second.

[4 marks]

c) Give a suitable reason why these models are not valid for all values of t.

[1 mark]

Q12 a) Prove the identity $\dfrac{\sin^4 x - \cos^4 x}{\cos^2 x} + \tan^2 x \equiv 2\tan^2 x - 1$

[3 marks]

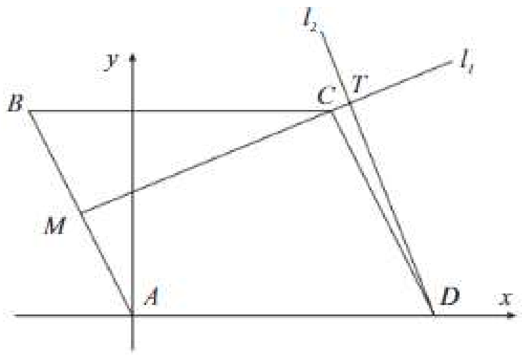
b) Hence, for $-180° \le x \le 180°$, solve the equation

$$\frac{\sin^4 x - \cos^4 x}{\cos^2 x} + \tan^2 x = 3\tan x - 1$$

Give your answers to one decimal place where appropriate.

[4 marks]

Q13 The points $A(0, 0)$, $B(-2, 4)$, $C(4, 4)$ and $D(6, 0)$ are the vertices of a parallelogram.
A line l_1 is drawn through C and the midpoint M of the side AB.
A second line l_2 is drawn through D and perpendicular to l_1.
T is the point of intersection of l_1 and l_2 as shown in the diagram.



a) Find the exact coordinates of the point T.

[6 marks]

b) Hence show that the points A, T and D are the vertices of an isosceles triangle.

[2 marks]

Q14 A closed cylinder made from a thin sheet of metal has radius r cm, length h cm and volume 128π cm³.

a) Show that $h = \dfrac{128}{r^2}$ and use this to find an expression for the surface area, S, of the cylinder in terms of r.

[2 marks]

b) Find the exact minimum value of the surface area of the cylinder and prove that the value is a minimum.

[5 marks]

Q15 The population of a rapidly expanding town is thought to be modelled by the equation $P = ab^t$, where t is measured in years and $t = 0$ corresponds to 1st January 1995. Some data relating to the population of the town is given in the table.

Year (1st January)	1995	2015
$\log_{10} P$	4.762	5.012

Using data from the table:

a) Write down an equation for $\log_{10} P$ in terms of t.

[3 marks]

b) Find the values of a and b, and the population predicted by the model for 1st January 2007 (to the nearest hundred).

[3 marks]

c) In which year did the population reach 100 000 according to the model?

[2 marks]

Q16 a) Explain why points $A(10, 4)$, $B(10, 10)$ and $C(2, 10)$ are the vertices of a right-angled triangle.

[1 mark]

b) Hence find the equation of the circle D_1 which passes through the points A, B and C.

[3 marks]

A second circle D_2 with the centre on the same vertical line as the centre of D_1 has equation $(x - 6)^2 + (y - 12)^2 = 10$. The circles D_1 and D_2 intersect at two distinct points.

c) Show that the point $M(9, 11)$ is a point of intersection of the circles and find the coordinates of the second point of intersection N.

[2 marks]

d) Show that the tangent lines to D_2 at the points M and N intersect on the circumference of the circle D_1.

[6 marks]

Pure Paper 2

1 Solve the simultaneous equations

$$2x + y = 3$$
$$x^2 + y^2 = 18$$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

2 Find the set of values of x for which

(a) $2(3x - 1) > 2 - 2x$

(2)

(b) $2x^2 - 11x + 12 > 0$

(4)

(c) both $2(3x - 1) > 2 - 2x$ and $2x^2 - 11x + 12 > 0$

(2)

3 Show that $\dfrac{3 + \sqrt{24}}{3 - \sqrt{6}}$ can be written in the form $a + b\sqrt{c}$ where a, b and c are integers.

(5)

4 Figure 1 shows a sketch of the curve $y = f(x)$. The curve passes through the y-axis at $(0, 7)$ and the x-axis at $(8, 0)$, has a minimum point at $(-4, 5)$ and a maximum point at $(1, 8)$.

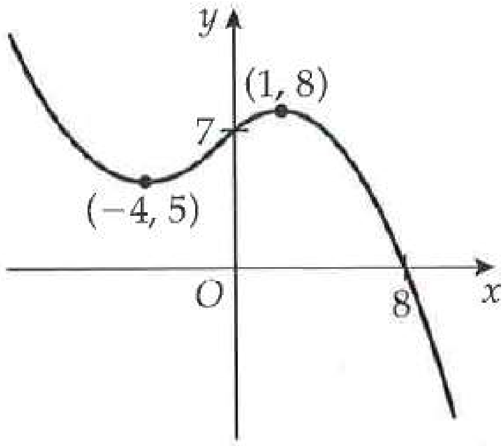


Figure 1

On separate diagrams, sketch the curves with the given equations, showing clearly the coordinates of any turning points, and the points at which the curve meets the coordinate axes.

(a) $y = f(x + 1)$ **(3)**

(b) $y = f(-x)$ **(3)**

(c) $y = \frac{1}{2}f(x)$ **(3)**

5 Relative to a fixed origin O, the point A has position vector $(\mathbf{i} + \mathbf{j})$, the point B has position vector $(2\mathbf{i} - 3\mathbf{j})$, and the point C has position vector $(4\mathbf{i} - 11\mathbf{j})$.

(a) Find the magnitude of the vector $\overrightarrow{OC}$. **(2)**

(b) Find the angle that $\overrightarrow{OB}$ makes with the vector $\mathbf{i}$. **(2)**

(c) Show that the points A, B and C are collinear. **(4)**

6 The curve C has equation

$$y = 2x^3 - 3x^2 + 7$$

Find the coordinates of the stationary points of the curve and determine the nature of each of them.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(7)

7 Solve the equation $3^x + 9^x = 6, x \in \mathbb{R}$

by using a substitution to find x.

(5)

8 (a) Use the factor theorem to show that $(x + 3)$ is a factor of

$$4x^3 + 8x^2 - 15x - 9$$

(2)

(b) Factorise $4x^3 + 8x^2 - 15x - 9$ completely.

(4)

9 Solve for $0 \leqslant \theta \leqslant 360°$, the equation

$$\cos 2\theta = \frac{\sqrt{3}}{2}$$

(3)

10 Given that $f(x) = 3x - 2 + \frac{4}{x^2}$

show that $\int_{\sqrt{3}}^{\sqrt{12}} f(x)\,dx = \frac{27}{2} - \frac{4\sqrt{3}}{3}$

(4

11 (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^5$$

giving each term in its simplest form.

(4)

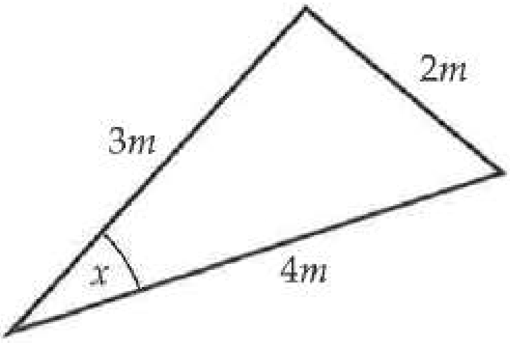
(b) Explain how you would use your expansion to give an estimate for the value of 1.94^5.

(1)

12 Prove that the roots of the equation $x^2 - 2px + p^2 - q^2 = 0$ where p and q are constants, are real.

(3)

13



Not drawn to scale

A student was asked to find the value of $\cos x$ in terms of m from the triangle in the diagram.

The student's attempt is shown below:

$$2m^2 = 4m^2 + 3m^2 - 2 \times 4m \times 3m\cos x$$

$$-5m^2 = -24m\cos x$$

$$\cos x = \frac{5m}{24}$$

(a) Identify the errors made by the student.

(2)

(b) Find the correct value of $\cos x$.

(3)

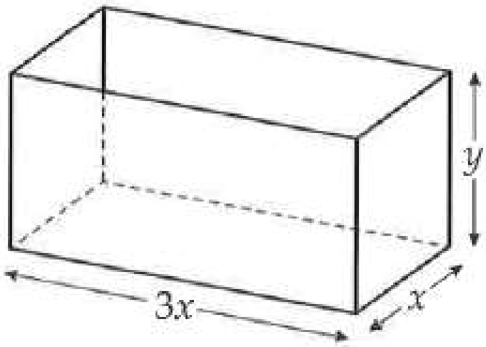
(c) Use this value to find an exact value for $\sin x$.

(2)

14 Prove, from first principles, that the derivative of $\frac{1}{x}$ is $-\frac{1}{x^2}$.

(4)

15 A hollow container in the shape of a cuboid has a base that measures $3x$ cm by x cm. The container doesn't have a top and has a height of y cm. The external surface area is $200\,\text{cm}^2$.



(a) Show that the volume, $V\,\text{cm}^3$, of the container is given by

$$V = 75x - \frac{9x^3}{8}$$

(4)

(b) Given that x can vary, find the maximum value of V to the nearest cm^3 and justify that this value is a maximum.

(7)

16 The total sales of a new novel in thousands of books, B, t months after release are modelled by the formula

$$B = 50(1 - e^{kt})$$

In the first month it is estimated that 10 000 books are sold.

(a) Find k.

(3)

(b) How many books do the publishers expect to sell in the first 5 months?

(2)

(c) Show that, according to this model, no more than 50 000 copies will be sold in the lifetime of the book.

(2)

(d) Show that the rate of sales of the book after t months is given by $\frac{dB}{dt} = Ae^{kt}$, where A is a constant to be found to 3 significant figures.

(2)

Pure Paper 3

1 (a) Express 27^{4k+2} in the form 9^m, giving m in terms of k.
(3)

(b) Find the value of n in the equation

$$\frac{8^{2n+1}}{4^n} = 32$$

(3)

2 $ax^2 + bx + c = 0$.

Prove, by completing the square, that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(6)

3 $f(x) = (2 + kx)^7$, where k is a constant.

Given that the coefficient of x^3 in the binomial expansion of $f(x)$ is 70, find the value of k.
(3)

4 (a) Show that the line $y = 2x + 11$ is a tangent to the circle

$$x^2 + y^2 - 6x - 4y = 32$$

(5)

(b) Find the equation of the diameter of the circle that passes through the point of contact of the tangent with the circle.
(3)

5 Prove that

$$\frac{1 - \sin\theta}{1 + \sin\theta} \equiv \left(\frac{1}{\cos\theta} - \tan\theta\right)^2$$

(5)

6 (a) On the same axes sketch the graphs

$$y = \frac{2}{x + 1}$$

$$y = 2x + 3$$

Show where the graphs cross the axes and any asymptotes.

(4)

(b) Find the coordinates of the points of intersection of the two graphs. Round your answers to 2 d.p.

(3)

7 (a) Given that $f(x) = \dfrac{(2x + 3)(x - 5)}{x}$, $x \neq 0$, find $f'(x)$.

(3)

(b) Show that $f(x)$ is increasing for the interval $1 < x < 2$

(2)

8 Factorise fully

$2x^5 - 32x$

(3)

9 Figures 1 and 2 show the curve with equation $y = 3x^2 + 2x - 1$

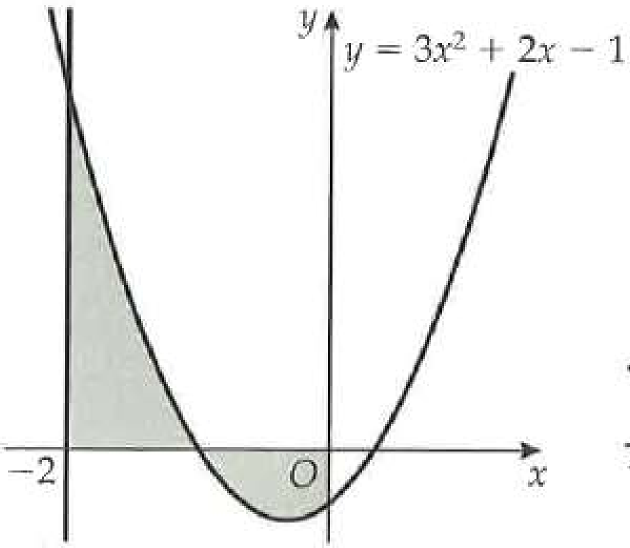


Figure 1

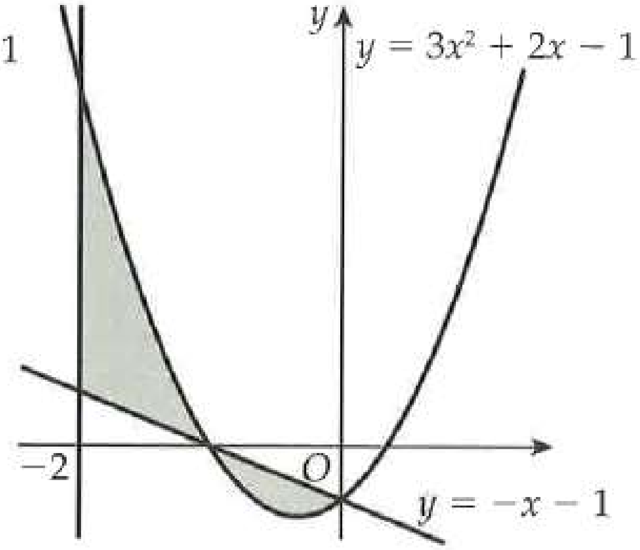


Figure 2

(a) Find the total shaded area shown in Figure 1, enclosed by the curve, the x-axis and the lines $x = -2$ and $x = 0$.

(4)

(b) Hence or otherwise find the shaded area shown in Figure 2 enclosed by the curve, the line $x + y = -1$ and the lines $x = -2$ and $x = 0$.

(3)

10 Prove that the sum of three consecutive multiples of 4 is always a multiple of 12.

(3)

11 (a) Find the equation of the straight line that passes through the points $A\ (-2, 7)$ and $B\ (3, 5)$, in the form $ax + by + c = 0$.

(3)

(b) Find the equation of the perpendicular bisector of AB.

(4)

(c) Point C lies on the perpendicular bisector of AB and is vertically above point B. Find the area of the triangle ABC.

(5)

12 $y = \int \left(2x^3 - 4 + \frac{1}{x^2}\right) dx$

(a) Given that $y = 0$ when $x = 2$, find y as a function of x.

(3)

(b) The point $P\ (-1, t)$ lies on the curve with equation $y = f(x)$. Find the value of t.

(2)

13 The value of Andy's car can be modelled by the formula

$$V = 18\,600e^{-0.23t} + k$$

where £V is the value of the car, t is the age in years and k is a positive constant.

(a) The value of the car when new was £19 550. Find the value of k.

(1)

(b) Find the value of the car after 3 years, to the nearest £.

(2)

(c) Find the rate of decrease in the value of the car in £ per year to the nearest £ at the instant when the car is 5 years old.

(3)

(d) Sketch the graph of V against t.

(2)

(e) Interpret the meaning of the value of k.

(1)

14 (a) Solve $5^{2x} = 8^{x-1}$ giving your answer to 3 significant figures.

(3)

(b) Solve $\log_3(x + 1) - \log_3(x) = 2$.

(3)

15 A parallelogram $ABCD$ has $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{AD} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{BD}\right|$.

(2)

(b) Find angle $B\hat{A}D$.

(3)

(c) Find the area of parallelogram $ABCD$.

(2)

16 The line L, defined by $y = 7x + k$, is a tangent to the curve C, defined by $y = ax^3 + x$, at the point where $x = 1$.
a and k are both constants.

(a) Find the value of a.

(2)

(b) Find the value of k.

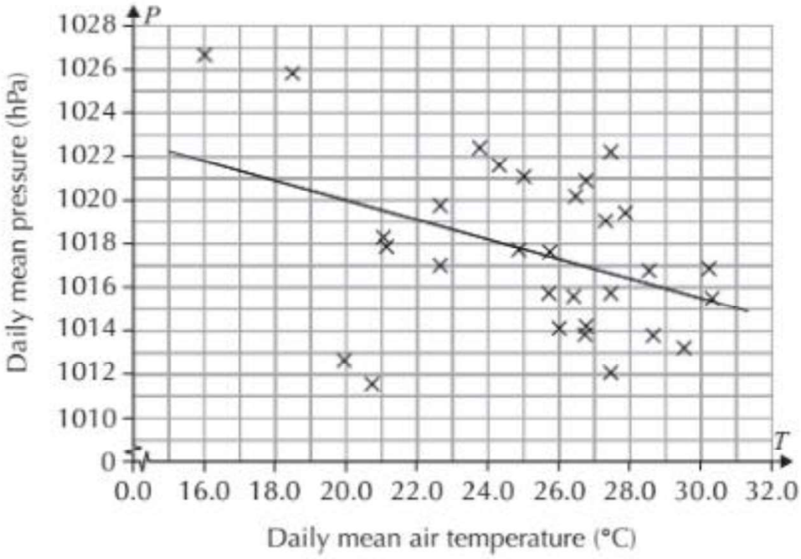
(2)

(c) Find the coordinates of the point where $y = 7x + k$ crosses the curve $y = ax^3 + x$.

(4)

Applied Paper 1

Q1 The scatter diagram below shows data from the large data set for Jacksonville for a random sample of days during May-October 1987. A regression line has been drawn for the data.



a) State: (i) the explanatory variable,
(ii) the response variable.

[1 mark]

b) Describe the correlation between daily mean air temperature and daily mean pressure shown on the diagram.

[1 mark]

The equation of the regression line of P on T is $P = -0.45T + 1029$.

c) Interpret the gradient of the regression line.

[1 mark]

d) (i) Use the equation of the regression line to predict the daily mean pressure for a daily mean temperature of 10 °C.

[1 mark]

(ii) Comment on the reliability of this prediction, giving a reason for your answer.

[1 mark]

Q2 Philippa is using the large data set to investigate daily total rainfall in Leuchars. She selects a random sample of size 6 from the May, June and July data for 2015. Readings are available for all dates from these months.
Philippa wishes to calculate the standard deviation of her sample data.

a) Using your knowledge of the large data set, suggest a difficulty that Philippa might face when calculating the standard deviation.

[1 mark]

Philippa codes her sample data using the coding $y = 10x - 2$, where y is the coded value and x is the original value for daily total rainfall (x mm). The coded values are:

4 0 224 6 152 20

b) Calculate:
(i) $\sum y$ (ii) $\sum y^2$

[2 marks]

c) Using your answers to b), calculate the mean and standard deviation of the coded data.

[2 marks]

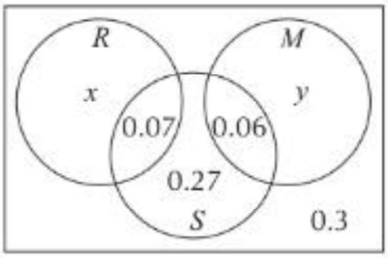
d) Find the standard deviation of Philippa's original data.

[1 mark]

Q3 A company employs interpreters who speak a range of languages.

R represents the event that an employee speaks Russian.
S represents the event that an employee speaks Spanish.
M represents the event that an employee speaks Mandarin.

The company wants to select an employee at random to complete a questionnaire. The Venn diagram on the right shows the probabilities of choosing an employee who speaks each language or combination of languages.



The events S and M are statistically independent.

a) Find the values of x and y.

[3 marks]

b) Find the probability that a randomly selected employee speaks Mandarin or Spanish, but not both.

[1 mark]

The company decides to send the questionnaire to several employees instead.
The employees are listed alphabetically and one of the first 10 is selected at random.
That employee and every tenth employee after them is selected to receive the questionnaire.

c) Give the name of the sampling method the company is using.

[1 mark]

Q4 A factory that produces springs selects a random sample of 80 springs. They use this sample to test the amount of force that can be applied to each spring before it breaks.
The table on the right summarises the data obtained from the sample.

Maximum force applied before breaking (x newtons)	Frequency
$0 \leq x < 5$	3
$5 \leq x < 10$	20
$10 \leq x < 15$	45
$15 \leq x < 20$	8
$20 \leq x < 25$	3
$25 \leq x < 30$	1

a) Find the class interval containing the median.

[2 marks]

b) Use linear interpolation to find an estimate for the median.

[3 marks]

c) Using the raw data it is found that $Q_1 = 9.4$ N and $Q_3 = 14.2$ N,
and that the single value in the class interval $25 \leq x < 30$ is 28.8 N.
Using $Q_3 + (1.5 \times \text{IQR})$ as the upper fence, show that the value 28.8 N is an outlier.

[1 mark]

Q5 A biologist inspects potato plants in a large field. The probability, p, that a given potato plant is infected with a certain fungus is thought to be 0.3. The biologist uses a binomial distribution and this probability to model the number of plants infected with the fungus.

a) If the biologist inspects 30 randomly-selected plants,
find the probability that at least 12 of these are infected with the fungus.

[2 marks]

The biologist suggests that the value of p has been underestimated. They select a random sample of 25 potato plants from the field and find that 10 of them are infected with the fungus.

b) Using a 5% significance level and clearly stating your hypotheses, find the critical region for a one-tailed hypothesis test to determine whether the value of p has been underestimated.

[3 marks]

c) Explain whether the biologist should reject the null hypothesis.
Interpret the outcome of this hypothesis test in context.

[2 marks]

The fungus is known to be easily passed from one plant to another.

d) Given the above information, explain whether a binomial distribution would be appropriate for modelling the infection of the potato plants.

[1 mark]

Q6 A particle P of mass m kg is acted upon by two forces, $\boldsymbol{F}_1 = \begin{pmatrix} 2.5 \\ -2 \end{pmatrix}$ N and $\boldsymbol{F}_2 = \begin{pmatrix} 9.5 \\ 5.5 \end{pmatrix}$ N.

a) Find the magnitude of the resultant force acting on P.

[3 marks]

b) Given that P accelerates at 2 ms^{-2}, find the value of m.

[2 marks]

Q7 A particle Q travels along a straight line such that the velocity, v ms^{-1}, of the particle at time t seconds is given by the equation $v = 1.4 - 0.9t + 0.1t^2$ for $t \geq 0$. Q is at the origin when $t = 0$.

a) Find the magnitude of the initial acceleration of Q.

[2 marks]

b) Find the times at which Q is instantaneously at rest.

[2 marks]

c) Find the total distance travelled by Q during the first 7 seconds.

[4 marks]

Q8 A stone, S, is projected vertically upwards from a level surface with initial speed 25 ms^{-1}. One second later another stone, T, is projected vertically upwards from the same point with initial speed 32 ms^{-1}. The two stones collide while in mid-air.

After projection, S and T move freely under gravity and are modelled as particles.

a) Find the height above the point of projection at which the stones collide and determine whether the stones are travelling in the same direction or opposite directions when they collide.

[7 marks]

b) Explain how the assumption that the stones move freely under gravity has been used in your calculations.

[1 mark]

Q9 A cage of mass 50 kg has a heavy load of mass 600 kg placed inside it. The cage and its load are initially at rest on a level surface before being raised vertically upwards by means of a light inextensible rope attached to the top of the cage. The cage accelerates upwards at 0.05 ms^{-2}.

a) Find the tension T in the rope.

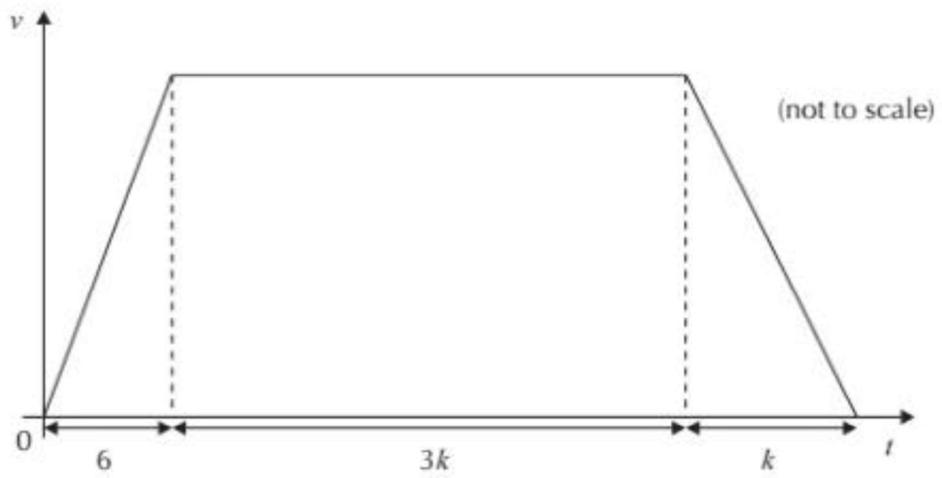
[2 marks]

b) (i) Find the normal reaction force R exerted by the load on the floor of the cage.

(ii) With reference to Newton's laws, explain how the size of the normal reaction force of the floor of the cage on the load compares to the value of R, found in part (i), as the cage accelerates upwards.

[3 marks]

A velocity time graph for the motion of the cage is shown below. After accelerating for 6 seconds the cage moves at a constant speed for $3k$ seconds before decelerating and coming to rest in a further k seconds.



c) The cage is raised a total of 9.3 m. With the aid of the graph, determine the deceleration of the cage as it comes to rest.

[4 marks]

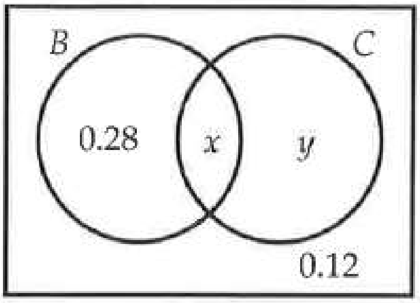
Applied paper 2

1 The Venn diagram shows the probabilities that a randomly chosen student at a college will take part in basketball and cricket.

B represents the event that a student takes part in basketball.

C represents the event that a student takes part in cricket.

x and y are probabilities.



The events B and C are statistically independent.

(a) Write down the probability that a student plays cricket.

(1)

(b) Find the values of x and y.

(4)

2 Lucy is investigating the daily minimum temperature for Leeming in the months of May to August (inclusive) 2015.

She uses the large data set for her investigation.

(a) Describe how Lucy can take a systematic sample of 30 days.

(3)

(b) From your knowledge of the large data set, explain why this might not give Lucy a sample of size 30.

(1)

The data Lucy collected are summarised as follows:

$$n = 30 \qquad \sum x = 248 \qquad \sum x^2 = 2361$$

(c) Calculate the mean and standard deviation of Lucy's sample.

(3)

3 A company requires applicants for jobs to take an aptitude test. The possible outcomes are: pass, fail, be retested. The probabilities for these outcomes are:

P(pass) = 0.3 P(fail) = 0.5 P(be retested) = 0.2

An applicant can be retested twice. If they do not pass after being retested twice, they are rejected.

The probabilities for the outcomes with the first retesting are:

P(pass) = 0.3 P(fail) = 0.5 P(be retested) = 0.2

The probabilities for the outcomes with the second retesting are:

P(pass) = 0.4 P(fail) = 0.6

(a) Draw a tree diagram to illustrate these outcomes.

(3)

(b) Find the probability that a randomly selected applicant is accepted.

(2)

(c) Three friends apply for jobs with this company. What is the probability that two of them pass at the first attempt and one of them at the final attempt?

(3)

4 A coffee shop provides its customers with wi-fi.

The probability that a randomly selected customer uses this wi-fi is 0.4. Twelve customers are selected at random.

(a) Give two reasons why the number of customers in the sample who use wi-fi can be modelled using a binomial distribution.

(2)

(b) Find the probability that at least six of the customers in the sample used the wi-fi.

(2)

Another branch of this coffee shop also provides its customers with free wi-fi. The manager suspects that the probability of a customer using this facility may be different from 0.4. A random sample of 20 customers is selected.

(c) Write down the hypotheses that should be used to test the manager's theory.

(1)

(d) Using a 10% level of significance, find the critical region for a two-tailed test to investigate the manager's theory. You should state the probability of rejection in each tail, which should be less than 0.05.

(3)

(e) Find the actual significance level of the test based on your critical region from part (d).

(1)

One morning, the manager finds that 11 out of 20 customers are using the wi-fi.

(f) Comment on the manager's theory in light of this observation.

(1)

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

5 A particle moves in a straight line. At time $t = 0$, the velocity is $u\,\text{m}\,\text{s}^{-1}$. The particle then accelerates with constant acceleration to $11\,\text{m}\,\text{s}^{-1}$ in 3 s.

The particle maintains this velocity of $11\,\text{m}\,\text{s}^{-1}$ for another 10 s.

It then decelerates with constant deceleration to rest in a further 2 s.

(a) Sketch a velocity–time graph to illustrate the motion of the particle.

(3)

(b) If the total distance travelled is 148 m, work out the value of u.

(3)

6 A particle moves along the x-axis with velocity $v\,\text{m}\,\text{s}^{-1}$.

At time t seconds, the velocity of the particle is given by

$$v = 35t - 10t^2$$

The positive direction is in the sense of increasing x.

Find the distance travelled between the times when the particle is instantaneously at rest.

(5)

7 A car is driven with constant acceleration, $a\,\mathrm{m\,s^{-2}}$, along a straight road.

Its speed when it passes a road sign is $u\,\mathrm{m\,s^{-1}}$.

In the first 3 seconds after passing the sign, the car travels 36 m.

5 seconds after passing the sign, the car has a speed of $26\,\mathrm{m\,s^{-1}}$.

(a) Write down two equations connecting a and u. Hence find the values of a and u.

(5)

(b) Find the total distance travelled by the car in the first 5 seconds after it passes the sign.

(2)

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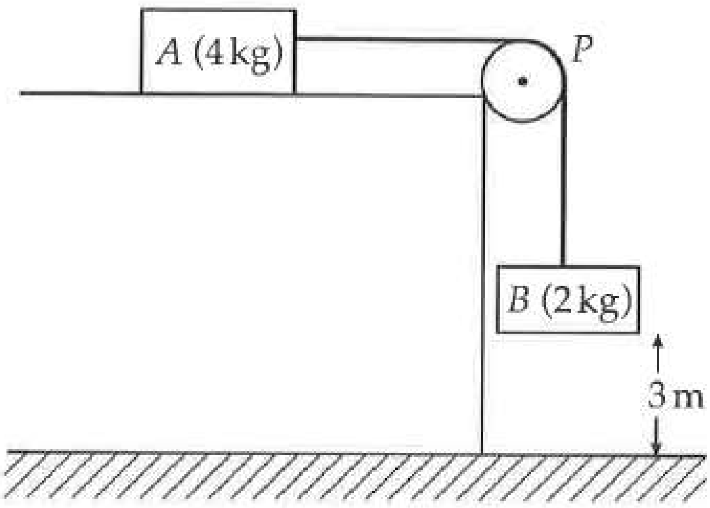
8 A block A of mass 4 kg is held at rest on a rough horizontal table.

It is attached to one end of a light inextensible string.

The string passes over a small smooth pulley P fixed at the edge of the table.



The other end of the string is attached to a block B of mass 2 kg, hanging freely below P and with B at a height of 3 m above the horizontal floor.

The system is released from rest with the string taut.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 14.8 N.

A and B are modelled as particles.

(a) Show that the acceleration of B is $0.8\,\mathrm{m\,s^{-2}}$.

(3)

(b) Calculate the tension in the string.

(2)

(c) Calculate the speed with which B hits the floor.

(2)

Block A never reaches the pulley at the edge of the table.

(d) After B has hit the floor, calculate the further distance A travels before coming to rest.

(4)

(e) State how, in your calculations, you have used the fact that the string is inextensible.

(1)

Applied paper 3

1 The discrete random variable Y can take only values 1, 2, 3, 4 and 5.

Y has probability function

$$P(Y = y) = \begin{cases} k(4 - y)^2, & y = 1, 2, 3 \\ k(3y - 8), & y = 4, 5 \end{cases}$$

where k is a constant.

(a) Find the value of k and construct a table giving the probability distribution of Y.

(4)

(b) Find $P(Y \geqslant 4)$

(1)

2 Thirteen students sat a test, marked out of 50. These are their scores:

33, 25, 36, 40, 14, 49, 31, 17, 31, 44, 35, 28, 34

(a) Write down the values of Q_1, Q_2 and Q_3 for this data.

(3)

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 or more than 1.5 times the IQR below Q_1 is called an outlier.

(b) Draw a box plot for this data.

(4)

3 A farmer collects data on annual rainfall, r cm, and the annual yield of broccoli, b tonnes per acre.

The table shows the results for 10 consecutive years.

r	75	79	78	84	74	76	80	85	77	81
b	4.5	4.8	4.7	5.1	4.6	4.6	4.9	5.2	4.7	4.8

The equation of the regression line of b on r is $b = 1.6 + 0.04r$.

(a) Give an interpretation of the value 0.04 in the regression equation.

(1)

(b) Comment on the reliability of using this regression equation to estimate the yield of broccoli in a year when the annual rainfall is

(i) 70 cm (ii) 82 cm

(2)

(c) Explain why the regression equation is not suitable to estimate the annual rainfall in a year when the yield of broccoli is 4.5 tonnes per acre.

(1)

4 In a survey, 120 shoppers in a supermarket were asked how many minutes, to the nearest minute, they had been in the store. The results are summarised in the table.

Number of minutes	Number of shoppers
1 – 4	5
5 – 9	20
10 – 19	28
20 – 29	51
30 – 59	16
Total	**120**

Use linear interpolation to estimate the 10% to 90% interpercentile range of this data.

(5)

5 All cars more than three years old have to undergo an annual MOT test. 20% of cars fail the MOT test because of problems with their lights. This is the most common cause of failure nationally. A group of 5 randomly selected cars undergo an MOT test.

(a) Give a reason why the binomial distribution is suitable for modelling the number of cars failing the MOT test due to a problem with their lights.

(1)

(b) Find the probability that exactly 2 of them fail the lights test.

(2)

A car hire firm has 15 cars taking their MOT test. Six of them fail the lights test.

(c) Using a 5% level of significance, find whether there is evidence to support the suggestion that cars from this firm fail the lights test more than the national average. You should state clearly the hypotheses that should be used.

(6)

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

6 A stone is thrown vertically upwards with speed $15\,\mathrm{m\,s^{-1}}$ from a point h metres above the ground.

The stone hits the ground 4 seconds later.

(a) Find the value of h.

(3)

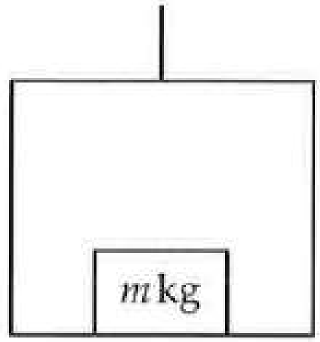
(b) State two modelling assumptions made when calculating your answer.

(2)

7 A lift of mass 220 kg is being lowered into a shaft by a vertical cable attached to the top of the lift.

The cable exerts an upward force of 2150 N on the lift.

A container of mass m kg rests on the floor of the lift.



There is a constant upward resistance of 160 N on the lift and the normal reaction between the container and the lift is of magnitude 480 N.

The lift descends with constant acceleration, $a\,\mathrm{m\,s^{-2}}$.

(a) Find the acceleration of the lift.

(4)

(b) Find the mass of the container.

(3)

8 Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle of mass 5 kg. The forces are given as:

$$\mathbf{F}_1 = 2a\mathbf{i} - 5b\mathbf{j}$$

$$\mathbf{F}_2 = -4b\mathbf{i} + 3a\mathbf{j}$$

$$\mathbf{F}_3 = 2\mathbf{i} - 2\mathbf{j}$$

where a and b are constants.

The particle is in equilibrium.

(a) Work out the values of a and b.

(3)

The force $\mathbf{F}_2$ is removed.

(b) Find the magnitude and bearing of the resulting acceleration of the particle.

(6)

9 A rabbit leaves its burrow at time $t = 0$ and runs alongside a straight fence before returning to its burrow.

The rabbit is modelled as a particle moving in a straight line.

The distance, s metres, of the rabbit from its burrow at time t seconds is given by

$$s = \frac{1}{5}(t^4 - 16t^3 + 64t^2)$$

where $0 \leqslant t \leqslant 8$

(a) Explain the restriction $0 \leqslant t \leqslant 8$

(3)

(b) Show that the rabbit is initially at rest and find its distance from the burrow when it next comes to instantaneous rest.

(6)