Mathematics Bridging work for year 12 into 13

Essential Course Materials – These should be purchased for September 2023

- Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS Textbook
  - *ISBN: 978-1292183398*
- Edexcel AS and A level Mathematics Statistics & Mechanics Year 1/AS Textbook
  - *ISBN: 978-1292232539*
- FX-991EX advanced scientific calculator

You will sit for a baseline assessment during your first lesson. Baseline assessment will be tested your bridging work. All work need to complete in an exercise book with clearly written title for each section and will be collected in your first lesson.

1.1 Index laws

1 Simplify these expressions:

a $b^3 \times b^4 \times b^2$ **b** $\frac{a^5}{a^3}$

c $5x^7 \times 3x^2$ **d** $(2x^2)^3 \div 4x^5$

Hint Use the laws of indices to simplify powers of the same base:

$a^m \times a^n = a^{m+n}$ $(a^m)^n = a^{mn}$

$a^m \div a^n = a^{m-n}$ $(ab)^n = a^n b^n$

2 Expand these expressions and simplify if possible:

a $x^3(4x^2 - 7) + 2x^5$ **b** $-5x^2(3 - 8x)$

c $x(3x + 4) - 7(5x - 2)$

Hint A minus sign before a bracket changes the signs of the terms inside the brackets.

3 Simplify these expressions:

a $\frac{x^9 - x^5 - x^7}{x^3}$ **b** $\frac{12x^3 + 8x^7}{4x}$

c $\frac{6x^4 + 12x^3 - (4x^5)^2}{2x^3}$

Hint Divide each term in the numerator by the denominator.

Remember that $a^1 = a$ and $a^0 = 1$

4 Simplify these expressions:

a $3x^4y^2 \times 7xy^3$ **b** $\frac{25x^{12}y^2}{5x^3y}$

Hint Simplify the numbers first and then use the laws of indices to simplify each letter.

E **5** Simplify these expressions:

a $\frac{27r^5s^7}{3r^3}$ **(2 marks)**

b $\frac{2(4a^4b^2)^3}{8ab^2}$ **(2 marks)**

E/P **6** **a** Given that $32 = 2^a$, find the value of a. **(1 mark)**

b Given that $81 = 3^b$, find the value of b. **(1 mark)**

E/P **7** Given that $\frac{33x^6 - 3(xy)^4 + (3x^2)^3}{6x^2}$ can be written in the form $10x^p - \frac{1}{2}x^qy^r$, find the values of p, q and r. **(3 marks)**

E **8** Write down the value of:

a $(125x^3)^{\frac{1}{3}}$ **(2 marks)**

b $\frac{20x^{\frac{5}{4}}}{5x^{\frac{1}{4}}}$ **(2 marks)**

c $(xy)^0$ **(1 mark)**

1.2 Expanding brackets

1 Expand these expressions and simplify if possible:

a $(x + 3)(x + 4)$ **b** $(x + 7)(x - 7)$

c $(x - 6)(x - 3)$

Hint Multiply each term in one expression by each term in the other expression and then simplify.

A minus sign in front of a term will change the sign of the term it is multiplied with.

2 Expand these expressions and simplify if possible:

a $(x + y)^2$ **b** $(x - 4)^2$ **c** $(2x - 5y)^2$

Hint $(a + b)^2 = (a + b)(a + b)$

3 Expand these expressions and simplify if possible:

a $(5x + 3y)(4x - 7y)$ **b** $(x - 9y)(x^2 - 1)$

c $(x + y)(x - 3y + 2)$

Hint Multiply each term in one expression by each term in the other expression and then simplify. A minus sign in front of a term will change the sign of the term it is multiplied with.

4 Expand these expressions and simplify if possible:

a $x(2x - 4y)(5x - 3y - 9)$

b $(3x - 2y)^3$

c $(2x - 1)(x + 3)(4x - 2)$

Hint You can expand three brackets by expanding and simplifying one pair of brackets first, then multiplying your expanded expression by every term in the third bracket.

(E) **5** Expand and simplify $(a + b)^3$ **(2 marks)**

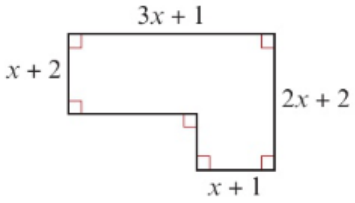
(E/P) **6** Given that $(2x - 7)(ax + b) = 6x^2 - 13x - 28$, find the values of the constants a and b. **(2 marks)**

(E) **7** The length of each edge of a cube is $(2x + 1)$ cm. Find an expression in terms of x for:

a the area of one face of the cube **(2 marks)**

b the volume of the cube. **(2 marks)**

(E/P) **8** The dimensions of a patio are shown, with lengths given in metres. The six edges are straight lines.



a Find an expression for the area of the patio in terms of x. **(3 marks)**

b Given that the area is 62 m^2, find the value of x. **(3 marks)**

(E/P) **9** Given that $(5a - b)(3a + 2b)(2a - b) = pa^3 - qa^2b - rab^2 + sb^3$, find the values of the constants p, q, r and s. **(2 marks)**

1.3 Factorising

1 Factorise these expressions completely:

a $6x + 18$ **b** $27x^2 - 9x$

c $x^3 - 4x^2$

Hint Find the highest common factor of the terms and write this factor outside the brackets. You can check your answers by expanding the brackets.

2 Factorise these expressions completely:

a $7ab^2 + 21a^2b^2$ **b** $8ab - 64b^3$

c $5ab^4 + 20a^3b^2c + 15b^5c^2$

Hint Find the highest common numerical factor, the highest common power of a and the highest common power of b. Then write these outside the brackets.

3 Factorise:

a $x^2 + 5x + 6$ **b** $x^2 - 3x - 10$

c $x^2 - 49$ **d** $y^2 - 16x^2$

Hint These are all quadratic expressions. They can be factorised into two brackets. For parts **c** and **d** you can use the rule for the difference of two squares: $x^2 - y^2 = (x + y)(x - y)$

4 Factorise completely:

a $3x^4 + 6x^3 - 18x^2$ **b** $4x^2 + 28x + 48$

c $2x^2 + x - 3$

Hint Start by writing part **b** as $4(x^2 + 7x + 12)$, then factorise the expression inside the brackets.

(E) **5** Factorise completely $x - 25x^3$ **(2 marks)**

(E) **6** Factorise completely $36x - 16x^3$ **(2 marks)**

(E) **7** Factorise completely $8x^3 + 20x^2 + 8x$ **(2 marks)**

(E/P) **8** Simplify $\frac{2x^3 + 6x^2 + 4x}{x^2 + 4x + 3}$ **(3**

1.4 Negative and fractional indices

1 Simplify:

a $x^{-4} \times x^{-1}$ **b** $\frac{x^5}{x^6}$ **c** $3x^{-10} \div 6x^3$

Hint Use the rules $a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$

2 Simplify:

a $2x^{\frac{1}{4}} \times 5x^{\frac{2}{3}}$ **b** $9x^{\frac{5}{2}} \div 3x^{\frac{2}{3}}$

c $(8x^3)^{\frac{2}{3}}$ **d** $7x^{-0.25} \times 2x^{0.5}$

Hint You can use the laws of indices with fractional powers.

3 Simplify:

a $\sqrt[5]{x} \times \sqrt{x}$ **b** $\sqrt[4]{x} \div (\sqrt{x})^3$ **c** $(\sqrt[3]{x})^2 \times (\sqrt{x})^7$

Hint Rewrite the questions with fractional powers using $a^{\frac{1}{m}} = \sqrt[m]{a}$ and $a^{\frac{n}{m}} = \sqrt[m]{a^n}$, and then simplify.

4 Evaluate:

a $16^{\frac{3}{2}}$ **b** $(-4)^{-3}$ **c** $\left(\frac{125}{27}\right)^{-\frac{2}{3}}$

Hint For parts **b** and **c**, you can use $a^{-m} = \frac{1}{a^m}$ You can use your calculator to evaluate negative and fractional powers.

5 Simplify:

a $\left(\frac{16}{25}x^3\right)^{\frac{5}{2}}$ **b** $\frac{x^5 + 8x^3}{x^{10}}$ **c** $\frac{30x^2 - 18x^5}{6x^7}$

Hint For part **a**, apply the power to both the fraction and x^3.

E **6** **a** Evaluate $64^{\frac{3}{2}}$ **(2 marks)**

b Simplify fully $x^3\left(2x^{\frac{1}{3}}\right)^6$ **(2 marks)**

E **7** **a** Find the value of $25^{-\frac{1}{2}}$ **(2 marks)**

b Simplify $x\left(\frac{3}{2}x^{-\frac{1}{4}}\right)^8$ **(2 marks)**

E/P **8** **a** Evaluate $81^{\frac{3}{4}}$, giving your answer as an integer. **(2 marks)**

b Simplify fully $\frac{\left(3x^{\frac{1}{2}}\right)^3}{9x^2}$ **(3 marks)**

E/P **9** Given that $y = \frac{1}{9}x^2$, express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{2}}$ **(2 marks)**

b $2y^{-1}$ **(2 marks)**

E/P **10** Express 25^{2x-5} in the form 5^y, where $y = ax + b$ for some constants a and b to be determined. **(2 marks)**

1.5 Surds

1 Simplify:

a $\sqrt{18}$ **b** $\sqrt{63}$ **c** $\sqrt{250}$

Hint You can manipulate surds using the rules $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

To simplify, look for factors of each number that are square numbers.

2 Simplify:

a $\frac{\sqrt{27}}{3}$ **b** $\frac{\sqrt{98}}{7}$ **c** $\frac{\sqrt{24}}{2}$

Hint Simplify the numerator first, then check whether the denominator is a factor of the numerator. If so, you can divide through by it.

3 Simplify:

a $4\sqrt{32} - 3\sqrt{8}$ **b** $\sqrt{75} + 2\sqrt{12} - \sqrt{27}$

c $\sqrt{200} + \sqrt{18} - 2\sqrt{72}$

Hint Simplify each surd, then collect like terms.

4 Expand and simplify if possible:

a $\sqrt{3}(\sqrt{27} - 1)$ **b** $(1 + \sqrt{2})(3 - 2\sqrt{2})$

c $(4 - \sqrt{3})(6 - \sqrt{7})$

Hint Multiply each term in one expression by each term in the other expression.

(E) **5** **a** Simplify $\sqrt{20} + \sqrt{45}$, giving your answer in the form $a\sqrt{b}$ where a and b are integers. **(2 marks)**

b Express $\sqrt{112}$ in the form $a\sqrt{7}$, where a is an integer. **(1 mark)**

(E/P) **6** Solve the equation $x - \sqrt{60} = 2\sqrt{3} - x$, giving your answer in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(2 marks)**

E 7 Expand and simplify:

a $(2-\sqrt{7})(\sqrt{7}-1)$ **(3 marks)**

b $(2\sqrt{7}+3)^2$ **(2 marks)**

E 8 Simplify:

a $(3\sqrt{11})^2$ **(1 mark)**

b $(7+\sqrt{3})(2-\sqrt{3})$ **(3 marks)**

E/P 9 Given that $243\sqrt{3} = 3^a$, find a. **(2 marks)**

1.6 Rationalising denominators

1 Simplify:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{35}{\sqrt{5}}$ **c** $\frac{9}{3\sqrt{3}}$

Hint For fractions in the form $\frac{1}{\sqrt{a}}$ multiply numerator and denominator by $\sqrt{a}$

2 Simplify:

a $\frac{\sqrt{7}}{\sqrt{21}}$ **b** $\frac{\sqrt{12}}{\sqrt{72}}$ **c** $\frac{\sqrt{75}}{\sqrt{125}}$

Hint Multiply numerator and denominator by the surd denominator and then simplify.

3 Rationalise the denominator and simplify:

a $\frac{1}{1+\sqrt{2}}$ **b** $\frac{2}{\sqrt{7}-1}$ **c** $\frac{3}{\sqrt{6}-\sqrt{5}}$

Hint For fractions in the form $\frac{1}{a+\sqrt{b}}$ multiply the numerator and denominator by $a-\sqrt{b}$

For fractions in the form $\frac{1}{a-\sqrt{b}}$ multip

numerator and denominator by $a+\sqrt{b}$

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4 Rationalise the denominator and simplify:

a $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ **b** $\frac{3-\sqrt{5}}{\sqrt{5}+5}$ **c** $\frac{3}{(2\sqrt{2}+3)^2}$

Hint For part **c**, start by expanding and simplifying the denominator.

E 5 Express $\frac{2}{2-\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers to be found. **(2 marks)**

E 6 Write each expression in the form $a+b\sqrt{5}$, where a and b are integers to be found:

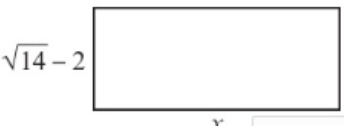
a $\frac{1+\sqrt{5}}{\sqrt{5}-2}$ **(3 marks)**

b $\frac{7+\sqrt{5}}{3+\sqrt{5}}$ **(3 marks)**

E/P 7 $\frac{3x^5-x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $3x^p - x^q$. Write down the values of p and q. **(2 marks)**

E/P 8 Show that $\frac{(7-2\sqrt{x})^2}{\sqrt{x}}$ can be written in the form $Ax^{-\frac{1}{2}} + Bx^{\frac{1}{2}} - C$ where A, B and C are constants to be found. **(3 marks)**

E/P 9 The diagram shows a rectangle with sides of length $(\sqrt{14}-2)$ cm and x cm. The area of the rectangle is 5 cm^2. Find x, giving your answer as simply as possible in surd form. **(3 marks)**



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Problem solving Set A

Bronze

a Simplify $\sqrt{147} - \sqrt{75}$ giving your answer in the form $a\sqrt{3}$, where a is an integer. **(2 marks)**

b Hence, or otherwise, simplify $\dfrac{24\sqrt{2}}{\sqrt{147} - \sqrt{75}}$ giving your answer in the form $b\sqrt{6}$, where b is an integer. **(2 marks)**

Silver

a Expand and simplify $(11 + \sqrt{5})(\sqrt{5} + 1)$ giving your answer in the form $a + b\sqrt{5}$, where a and b are integers. **(2 marks)**

b Hence, or otherwise, simplify $\dfrac{11 + \sqrt{5}}{\sqrt{5} - 1}$ giving your answer in the form $c + d\sqrt{5}$, where c and d are integers. **(2 marks)**

Gold

Simplify $\dfrac{6\sqrt{3} - 4}{5 - \sqrt{3}}$, giving your answer in the form $p\sqrt{3} - q$, where p and q are positive rational numbers. **(4 marks)**

Problem solving Set B

Bronze

a Express 4^{x+2} in the form 2^y, stating y in terms of x. **(2 marks)**

b Hence, or otherwise, solve the equation $4^{x+2} = 32$. **(2 marks)**

Silver

a Rewrite the equation $(4^{x-1})^2 - 6(4^{x-1}) + 8 = 0$ in the form $y^2 + by + c = 0$, where $y = 4^{x-1}$ and b and c are constants to be found. **(1 mark)**

b Factorise your equation from part **a** and hence, or otherwise, solve for x. You must show each step of your working. **(3 marks)**

Gold

Solve the following equation, showing each step of your working:

$(9^{x-1})^2 - 30(9^{x-1}) + 81 = 0$ **(5 marks)**

2.1 Solving quadratic equations

1 Solve the following equations using factorisation:

a $x^2 + 7x + 10 = 0$ **b** $x^2 - 5x - 24 = 0$

c $x^2 + 6x = 0$

Hint The factorised quadratic equation $(x - a)(x - b) = 0$ will have solutions $x = a$ and $x = b$.

2 Solve the following equations using factorisation:

a $7x^2 = 21x$ **b** $4x^2 = 49$

c $4x^2 - 20x + 25 = 0$ **d** $2x^2 - 5x + 2 = 0$

Hint Write the equation in the form $ax^2 + bx + c = 0$ before factorising.

Solutions to quadratic equations do not have to be integers.

3 Solve the following equations:

a $(x - 4)^2 = 0$ **b** $x(2x - 1) = 21$

c $4x^2 + 4x + 24 = 2x^2 - 10x$

Hint A quadratic equation may have only one solution, called a repeated root: If $(x - a)^2 = 0$, then $x = a$.

4 Solve the following equations using the quadratic formula.

Give your answers exactly, leaving them in surd form.

a $x^2 + 8x + 6 = 0$ **b** $x^2 + 4x = 1$ **c** $2x^2 - 12x + 15 = 0$

Hint The solutions to $ax^2 + bx + c = 0$ are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

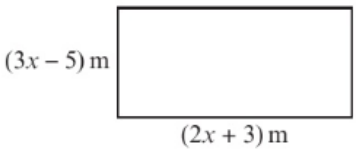
5 Solve the following equations using a suitable method.
Where necessary, give your answers to 3 significant figures:

a $10x - x^2 - 9 = 0$ **b** $64x^2 = 100$ **c** $3x^2 + 10x - 25 = 0$

d $12x = 3x^2 + 5$ **e** $x = \sqrt{5x}$ **f** $2x^2 + 6x + 1 = 0$

E **6** The length of a rectangular carpet is $(2x + 3)$ metres and its width is $(3x - 5)$ metres.

The carpet has an area of 20 m^2.



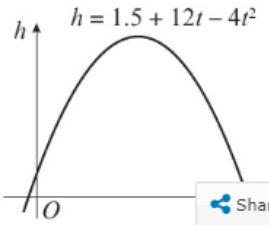
a Show that $6x^2 - x - 35 = 0$. **(2 marks)**

b Hence find the length and width of the carpet, in metres. **(2 marks)**

E **7** Solve the equation $x^2 + 4x + 1 = 0$. Write your answer in the form $a \pm \sqrt{b}$, where a and b are integers to be found. **(2 marks)**

E **8** Solve the equation $0.1x^2 + 1.6x = 0.8$. Give your answers to 3 significant figures. **(2 marks)**

E/P **9** The height, h metres, of a ball at time t seconds after it is thrown up in the air can be modelled by the equation $h = 1.5 + 12t - 4t^2$.



a Find how many seconds it takes for the ball to hit the ground again. Give your answer correct to 3 significant figures. **(2 marks)**

b Find the times when the height of the ball is 9.5 m. **(2 marks)**

2.2 Completing the square

1 Complete the square for these expressions:

a $x^2 + 2x$ **b** $x^2 - 8x$

c $x^2 - 12x$

Hint $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

2 Complete the square for these expressions:

a $x^2 - 8x + 12$ **b** $x^2 - x - 12$

c $x^2 - 3x - 4$

Hint When a quadratic expression is in completed square form, x only appears once. The constants in your answer do not have to be integers.

3 Write each of these expressions in the form $a(x + b)^2 + c$, where a, b and c are constants to be found:

a $3x^2 - 12x + 17$ **b** $5x^2 - 10x + 12$ **c** $3x^2 - 7x + 2$

Hint Start by factorising the first two terms of each expression.

4 Solve these quadratic equations by completing the square. Leave your answers in surd form.

a $x^2 - 2x - 10 = 0$ **b** $5x^2 + 12x + 6 = 0$

c $3x^2 - 7x - 2 = 0$

Hint Write the left-hand side of each equation in completed square form. Then use inverse operations to make x the subject of the equation. Remember to include ± when you take square roots of both sides of the equation.

E/P **5** $f(x) = x^2 - 7x - 2$, $x \in \mathbb{R}$

a Express $f(x)$ in the form $(x - a)^2 - b$, where a and b are constants. **(2 marks)**

b Hence write down the minimum value of $f(x)$. **(1 mark)**

(E) **6** $h(x) = 4 - 2x - 3x^2$, $x \in \mathbb{R}$

a Express $h(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. **(2 marks)**

b Hence, or otherwise, find the exact solutions to $h(x) = 0$. **(2 marks)**

(E) **7** $6x - 2 - x^2 \equiv q - (x + p)^2$, where p and q are integers.

a Find the value of p and the value of q. **(3 marks)**

b Hence, or otherwise, solve the equation $6x - 2 - x^2 = 0$. **(2 marks)**

2.4 Quadratic graphs

1 Sketch graphs of each of the following equations, showing the coordinates of the points where the graph crosses the coordinate axes.

a $y = x^2 + 11x + 18$ **b** $y = 4x^2 - 16$

c $y = -6x^2 + 2x$

Hint Factorise each equation to find the points where $y = 0$. These are the values of x at the points where the graph crosses the x-axis. To find the y-intercept, substitute $x = 0$ into the equation.

2 Find the coordinates of the turning point on each of these graphs:

a $y = (x - 1)^2 + 9$ **b** $y = x^2 + x - 6$

c $y = -x^2 - 13x - 42$

Hint You can find coordinates of the turning point on a quadratic curve by completing the square. The curve with equation $y = (x - a)^2 + b$ will have a turning point at (a, b).

3 Sketch the graphs of the following equations. For each graph, indicate where the graph crosses the coordinate axes, and write down the coordinates of the turning point and the equation of the line of symmetry.

a $y = x^2 - 6x + 20$ **b** $y = -2x^2 - 5x - 2$ **c** $4x^2 - y = 4x + 3$

Hint A quadratic graph has a vertical line of symmetry that passes through its turning point.

4 Sketch the graphs of the following equations. For each graph, indicate where the graph crosses the coordinate axes, leaving your answer in surd form. Write down the coordinates of the turning point and the equation of the line of symmetry.

a $y = x^2 + 7x + 5$ **b** $y = -5x^2 - 12x - 3$ **c** $y = 2x^2 + 7x + 4$

Hint After drawing your axes, write in all the required coordinate points and then draw a smooth curve through these points.

(E) **5** The expression $8x - 7 - x^2$ can be written in the form $q - (x - p)^2$, where p and q are integers.

a Find the value of p and the value of q. **(3 marks)**

b Sketch the curve with equation $y = 8x - 7 - x^2$, showing clearly the coordinates of any points where the curve crosses the coordinate axes. **(3 marks)**

E **6** $f(x) = x^2 + 6x + 4,\ x \in \mathbb{R}$

a Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants. **(2 marks)**

The curve C with equation $y = f(x)$ crosses the y-axis at point P and has a minimum point at the point Q.

b Sketch the graph of C, showing the coordinates of points P and Q. **(3 marks)**

c Explain why the equation $f(x) = -6$ has no real solutions. **(1 mark)**

E/P **7** $p(x) = 3 - 2x,\ q(x) = x^2 - 9x - 10,\ x \in \mathbb{R}$

a Solve the equation $q(x) = 0$. **(2 marks)**

b Sketch the graphs of $y = p(x)$ and $y = q(x)$ on the same set of axes.
Label all points where the curves intersect the coordinate axes. **(4 marks)**

E/P **8** The graph of $y = ax^2 + bx + c$ has a minimum at $(2, -5)$ and passes through $(3, 0)$.
Find the values of a, b and c. **(4 marks)**

3.1 Linear simultaneous equations

1 Solve these simultaneous equations by elimination:

a $6x + y = 9$
$4x - y = 11$

b $2x + 3y = 8$
$3x + 2y = 7$

c $4x - 3y = 2$
$5x - 7y = 9$

Hint Either the x- or y-coefficients in the pair of equations need to have the same value.

You can then add or subtract the equations to eliminate one variable and solve for the other.

You need to find the value of both variables.

2 Solve these simultaneous equations by substitution:

a $5x - 2y = 3$
$x + 4y = 5$

b $2x + 5y = 37$
$y = 11 - 2x$

c $4x + 3y = 5$
$2x - 6y = -5$

Hint Rearrange one equation to make x or y the subject and then substitute this expression into the other equation and solve.

3 Solve these simultaneous equations:

a $3(x - y) + 6 = 0$
$y + x = 8$

b $5(x - 1) = -2y$
$3x - 29 = 4y$

c $\frac{4x - y}{2} = 11$
$\frac{5 - 3x}{5} = y$

Hint Rearrange the equations into the form $ax + by = c$, where a, b and c are constants, and then solve using either elimination or substitution.

(E) **4** Solve the simultaneous equations

$x + y = 2$
$2y = 18x - 6$

(2 marks)

(E) **5** Solve the simultaneous equations

$3x - y = -5$
$0.5y + 2x = 4$

giving your answers in exact form. **(2 marks)**

(E/P) **6** $6ky + 9x = 12$
$ky - x = 4.5$

are simultaneous equations where k is a constant.

a Show that $x = -1$. **(2 marks)**

b Given that $y = 7$, find the value of k. **(1 mark)**

(E/P) **7** Two students are attempting to solve the simultaneous equations

$4x + 6y = 10$
$2x = 5 - 3y$

Ben says that these equations have no solutions, and Nisha says that they have infinitely many solutions. Who is correct? Explain your answer. **(2 marks)**

3.2 Quadratic simultaneous equations

1 Solve the simultaneous equations:

a $xy = 64$
$4x - y = 60$

b $x^2 + y^2 = 10$
$x + y = 4$

c $x - y + 1 = 0$
$3x^2 - 4y = 0$

Hint A quadratic equation can contain terms involving xy, x^2 and y^2.
Rearrange the linear equation to make x or y the subject, then substitute this expression into the quadratic equation and solve.

2 Solve the simultaneous equations:

a $y - x^2 + 3x + 2 = 0$
$y - 2x + 6 = 0$

b $x^2 + 2x - y = 14$
$y + 2 = x$

c $x^2 + y^2 = 5$
$y = 5 - 3x$

Hint Each set of equations will have two pairs of solutions.
You need to identify the solutions and pair them up correctly.

3 Solve these simultaneous equations, giving your answers to 2 decimal places:

a $3x - 7 = y$
$x^2 - 3x - 2 = 2y$

b $2x^2 - xy + y^2 = 8$
$x + y = 1$

c $y^2 - 5x^2 = 20$
$4x - 7 = y$

(E) **4** Solve the simultaneous equations

$$x + y = 4$$
$$4y^2 - x^2 = 12$$

(4 marks)

(E) **5** Solve the simultaneous equations

$$y + 2x + 1 = 0$$
$$y^2 + 3x^2 + 2x = 0$$

(4 marks)

(E/P) **6**

$$mx - y - 2 = 0$$
$$x^2 - 2x + y^2 - 4y = 4$$

where m is a real constant.

Given that these simultaneous equations have exactly one pair of solutions, find the two possible values of m, giving your answers in exact form. **(7 marks)**

(E/P) **7** The values of x and y satisfy the simultaneous equations

$$y - 2x = 8$$
$$x^2 + 2ky + 4k = 0$$

where k is a non-zero constant.

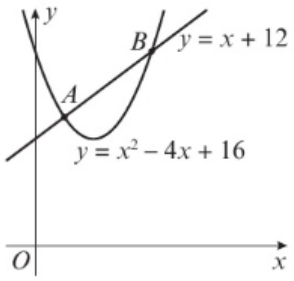
a Show that $x^2 + 4kx + 20k = 0$. **(2 marks)**

b Given that $x^2 + 4kx + 20k = 0$ has equal roots, find the value of k. **(3 marks)**

c For the value of k found in part **b**, solve the simultaneous equations. **(3 marks)**

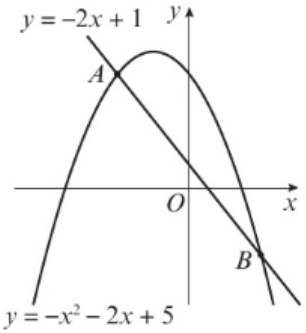
E **4** The diagram shows the curve with equation $y = x^2 - 4x + 16$ and the line with equation $y = x + 12$. The curve intersects the line at points A and B.

Using an appropriate algebraic method, find the coordinates of A and B. **(4 marks)**



E **5** The diagram shows part of curve with equation $y = -x^2 - 2x + 5$ and part of the line with equation $y = -2x + 1$. The curve intersects the line at points A and B.

Using an appropriate algebraic method, find the coordinates of A and B. **(4 marks)**



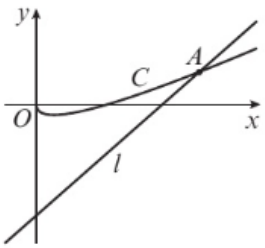
E/P **6** $p(x) = 1 - \frac{1}{2}x$, $q(x) = x^2 - 4x - 10$

a Sketch the graphs of $y = p(x)$ and $y = q(x)$ on the same set of axes. Label all points where the graphs cut the coordinate axes. **(4 marks)**

b Use an algebraic method to find the coordinates of any points of intersection of the graphs $y = p(x)$ and $y = q(x)$. **(4 marks)**

E/P **7** The diagram shows a sketch of the curve C with equation $y = 2x - 3\sqrt{x}$ $(x \geqslant 0)$, and the line l with equation $y = 3x - 12$.

The line cuts the curve at point A as shown in the diagram. Using algebra, find the x-coordinate of point A. **(5 marks)**



E/P **8** $f(x) = 6 - 5x$ and $g(x) = 4 - 0.5x$

a Use an algebraic method to find the exact coordinates of the point of intersection of the graphs of $y = f(x)$ and $y = g(x)$. **(3 marks)**

b Sketch these graphs on the same set of axes, showing all intersections with the coordinate axes. **(2 marks)**

3.4 Linear inequalities

1 Find the set of values of x for which:

a $x + 4 \geqslant 12$ **b** $7 - 5x < 62$

c $3(3 - 2x) \leqslant 2(3 + 2x)$

Hint You can solve inequalities like equations. However, if you divide both sides by a negative number, you need to reverse the inequality sign.

2 Find the set of values of x for which:

a $7 \leqslant 5x - 3 \leqslant 17$ **b** $\frac{x}{3} - \frac{2x}{9} < 3$

c $3x(3 + x) + x^2 \geqslant 1 + x(6 + 4x)$

Hint For part **c**, expand the brackets and simplify before solving.

3 Use set notation to describe the set of values of x for which:

a $2(1 + x) < 4 - x$ and $x - 3 < 4x + 6$

b $\frac{x}{4} + 3 \leqslant 6$ and $2(3x - 5) \geqslant 20$

c $0.5(4x + 3) < 2.5$ or $\frac{x - 3}{5} > 7$

Hint In set notation $x < 3$ and $x > -1$ is written as $\{x : -1 < x < 3\}$, or alternatively $\{x : x > -1\} \cap \{x : x < 3\}$

(E) **4** Find the set of values of x for which:

a $2x - 6 > 3 - x$ **(2 marks)**

b $4x - 5 < 3 + 2x$ **(2**

(E) **5** Use set notation to write the set of values of x for which:

a $20 - 2x > 15 - 7x$ **(2 marks)**

b $x - 5 \leqslant 3 - 9x$ **(2 marks)**

(E) **6** Use set notation to write the set of values of x for which:

a $3(3x + 4) \geqslant 2 - x$ **(2 marks)**

b $7(x - 3) < 3(x - 3)$ **(2 marks)**

(E/P) **7** The width of a rectangular field is x metres, $x > 0$. The length of the pitch is 30 m more than its width. Given that the perimeter of the pitch must be less than 400 m,

a form a linear inequality in x **(2 marks)**

b solve your inequality. **(2 marks)**

(E/P) **8** Use set notation to describe the set of values of x for which:

a $3(2 + x) \geqslant 2 - x$ and $x + 7 > 5x - 1$ **(4 marks)**

b $0.25(8x + 4) < 5.5$ or $\frac{x - 2}{9} > 11$ **(4 marks)**

5.1 $y = mx + c$

1 Work out the gradients of the lines joining these pairs of points:

a (4, 3), (8, 6) **b** (5, 2), (7, −1)

c $(3p, -4p), (8p, -2p)$

Hint The gradient, m, of a line joining the points with coordinates (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

2 The line l has gradient $\frac{1}{3}$ and passes through (0, 7). Find an equation for l in the form

a $y = mx + c$ **b** $px + qy + r = 0$

(P) **3** The points $A(1, 2)$, $B(4, 1)$ and $C(12, k)$ are collinear. Work out the exact value of k.

Hint A, B and C are **collinear** if they all lie on the same straight line.

This means that the gradients of the line segments AB, BC and AC are equal.

4 For the line with equation $4x - 5y + 12 = 0$, find:

a the gradient

b the coordinates of the y-intercept

c the coordinates of the x-intercept.

Hint The equation of a straight line can be written in the forms:

- $y = mx + c$, where m is the gradient and c is the y-intercept
- $ax + by + c = 0$, where a, b and c are integers.

(E/P) **5** The gradient of the line joining the points $(2, 3a)$ and $(5a, -2)$ is −1. Work out the value of a. **(2 marks)**

(E/P) **6** The line l_1 with gradient $-\frac{1}{5}$ passes through (0, 3). l_1 intersects the line l_2 with equation $3x - 4y + 7 = 0$ at point P. Find the exact coordinates of P. **(4 marks)**

(E/P) **7** The points $A(-3p - 3, 2p)$, $B(-5p + 1, 0)$ and $C(0, 8p)$, where p is a constant, $p \neq 0$, are collinear. Find:

a the value of p **(4 marks)**

b the gradient of the line through A, B and C. **(2 marks)**

(E/P) **8** The line l_1 has gradient $\frac{2}{3}$ and passes through the point (0, −4).

a Find an equation of l_1 in the form $ax + by + c = 0$. **(3 marks)**

The line l_2 with equation $3x - ky + 25 = 0$ intersects l_1 at the point $(p, -2)$ where k and p are constants. Find:

b the value of p **(2 marks)**

c the value of k.

5.2 Equations of straight lines

1 Find the equation of the line with gradient m that passes though the point (x_1, y_1) when:

a $m = -2$ and $(x_1, y_1) = (4, -5)$

b $m = \frac{1}{4}$ and $(x_1, y_1) = (-2, 6)$

c $m = -\frac{1}{8}$ and $(x_1, y_1) = (-3, -2)$

d $m = -\frac{3}{5}$ and $(x_1, y_1) = (1, -3)$

Hint The equation of the line with gradient m that passes though the point (x_1, y_1) can be found using $y - y_1 = m(x - x_1)$

2 Find the equations of the lines that pass through these pairs of points:

a (4, −3) and (−6, 9)

b (−1, 3) and (5, −2)

c $\left(\frac{1}{2}, -\frac{1}{3}\right)$ and $\left(-\frac{3}{4}, \frac{2}{3}\right)$

Hint To find the equation of the line that passes through the points (x_1, y_1) and (x_2, y_2), first find the gradient m using $m = \frac{y_2 - y_1}{x_2 - x_1}$, then use $y - y_1 = m(x - x_1)$

3 The line that passes through the points (1, −4) and (−3, 6) meets the x-axis at the point P. Work out the coordinates of P.

Hint Find the equation of the line, then substitute in $y = 0$ and solve to find x.

4 The line that passes through the points (−2, 5) and (4, −3) meets the y-axis at the point Q. Work out the exact coordinates of Q.

(E) **5** The line l passes through the points $A(3, -6)$ and $B(-2, -10)$.
Find an equation for l, giving your answer in the form $y = mx + c$. **(3 marks)**

(E) **6** The points $A(-6, 3)$ and $B(15, -4)$ lie on the line L.
Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

(E/P) **7** The line l_1 passes through the point (6, −3) and has gradient $\frac{1}{3}$. l_1 meets the line l_2 with equation $x + 2y = 10$ at the point P. Calculate the coordinates of P. **(4 marks)**

(E/P) **8** The line l_1 passes through the point (5, −4) and has gradient $\frac{1}{4}$

a Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

The line l_2 passes through the origin O and has gradient −5. The lines l_1 and l_2 intersect at the point P.

b Calculate the coordinates of P. **(4 marks)**

5.3 Parallel and perpendicular lines

1 Work out whether these pairs of lines are parallel.

Hint Parallel lines have the same gradient.

a $2y = 3x - 5$
$6x - 4y + 11 = 0$

b $3x - 4y + 9 = 0$
$9x + 12y - 10 = 0$

c $5x + 2y - 15 = 0$
$10x + 4y + 9 = 0$

2 Work out whether each of these pairs of lines are perpendicular.

a $3y = 2x + 7$
$4x + 6y + 1 = 0$

b $5x - 3y + 2 = 0$
$5y = 3x + 6$

c $4x - y - 5 = 0$
$2x + 8y - 15 = 0$

Hint If two lines are perpendicular, the product of their gradients is -1.
If a line l has gradient m, the gradient of a line perpendicular to l is $-\frac{1}{m}$

3 Find an equation of the line that passes through the point $(-3, 5)$ and is parallel to the line $2x - 3y + 7 = 0$.

4 Find an equation of the line that passes through the origin and is parallel to the line joining the points $(1, -6)$ and $(-2, 9)$.

E/P **5** The line l_1 has equation $4y - 8 = 3x$. The point P with x-coordinate 4 lies on l_1.
The line l_2 is perpendicular to l_1 and passes through the point P. Find an equation of l_2, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4 marks)**

E/P **6** The line L has equation $2y = 1 - 3x$.

a Show that the point $P(3, -4)$ lies on L. **(1 mark)**

b Find an equation of the line perpendicular to L, which passes through P.

5.4 Length and area

1 Find the exact distance between these pairs of points:

a $(1, -2), (-5, 6)$

b $(-8, 4), (16, -3)$

c $(6, -5), (-10, -1)$

Hint You can find the distance d between the points (x_1, y_1) and (x_2, y_2) by using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
If the number inside the square root is not a perfect square, write the surd in its simplest form.
← Section 1.5

2 Consider the points $A(-2, 5)$, $B(3, 1)$ and $C(8, -3)$. Determine whether the line segment joining the points A and B is congruent to the line segment joining the points B and C.

Hint Line segments are congruent if they are the same length.

3 The vertices of a triangle are $P(-3, 2)$, $Q(2, 5)$ and $R(4, 2)$.
Find the area of triangle PQR.

Hint Sketch the triangle and label its vertices.
Use area $= \frac{1}{2} \times$ base $\times$ perpendicular height.

P **4** The distance between the points $(-2, 11)$ and $(x, 8)$ is $\sqrt{58}$.
Find the two possible values of x.

Hint Use the formula for the distance between two points to form a quadratic equation, then solve this equation.
← Section 2.1