## Dear Year 12 Mathematicians,

Well done for the work you have put in during this period of uncertain times, and your excellent engagement with the live lessons. We have begun the Year 2 Pure content, and in preparation for what is to come in Year 13, you need to prepare by completing the tasks in this pack.

The tasks have the purpose of recapping Year 1 content for both Pure and Applied, as well as being the foundations for the Year 2 content that you are yet to learn. Mastering the Year 1 content will allow you to be in the strongest possible position in Year 13.

## Task 1 - Mastering Pure Year 1 (starts on page 2)

Task 2 - Mastering Applied Statistics Year 1 (starts on page 72)
Task 3 - Mastering Applied Mechanics Year 1 (starts on page 96)
Each week you should complete the following:

- From Task 1 - any 2 of the Pure topic-based questions
- From Task 2 - at least 1 of the Statistics topic-based questions
- From Task 3 - at least 1 of the Mechanics topic-based questions

There are a lot of questions for Pure, so as a minimum, the below must be completed:

| Algebra 1 | Page 3 | All even questions |
| :--- | :--- | :--- |
| Algebra 2 | Page 9 | All even questions |
| Binomial Expansion | Page 15 | All odd questions |
| Coordinate Geometry | Page 20 | All even questions |
| Differentiation 1 | Page 29 | All odd questions |
| Differentiation 2 | Page 37 | All odd questions |
| Exponentials and Logarithms | Page 44 | All even questions |
| Graphs and Transformations | Page 49 | All odd questions |
| Integration | Page 59 | All odd questions |
| Trigonometry | Page 68 | All questions |

Please complete all your work on paper, or feel free to print out the pack and work directly on there. Your work should be completed, ready to fully submit via email, or the Teams Assignment to your class teachers at the start of the September term.

If you need a recap of any Year 1 teaching for either Pure or Applied, there are fantastic videos you can view to support you with these tasks on YouTube as follows:
https://www.youtube.com/channel/UCyyRmnmtgVy5Sm7 UiCLFgQ/playlists?view=50\&sort =dd\&shelf id=9

Thank you for all your hard work and efforts this year. It has been a pleasure teaching you all and we look forward to seeing you soon.

## Mrs Bayar

$$
\begin{gathered}
\text { Task } 1 \text { - } \\
\text { Mastering } \\
\text { Pure Year } 1
\end{gathered}
$$

## Write your name here



## AS and A level Mathematics

## Practice Paper

Pure Mathematics - Algebra (part 1)

## You must have: <br> Mathematical Formulae and Statistical Tables (Pink)

Total Marks

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.


## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 17 questions in this question paper. The total mark for this paper is 80 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1*. Show that $\frac{2}{\sqrt{12-\sqrt{ } 8}}$ can be written in the form $\sqrt{ } a+\sqrt{ } b$, where $a$ and $b$ are integers.

2*. (a) Simplify

$$
\sqrt{50}-\sqrt{18}
$$

giving your answer in the form $a \sqrt{2}$, where $a$ is an integer.
(b) Hence, or otherwise, simplify

$$
\frac{12 \sqrt{3}}{\sqrt{50}-\sqrt{18}}
$$

giving your answer in the form $b \sqrt{c}$, where $b$ and $c$ are integers and $b \neq 1$

3*. (a) Write down the value of $32^{\frac{1}{5}}$
(b) Simplify fully $\left(32 x^{5}\right)^{-\frac{2}{5}}$

4*. (a) Evaluate $81^{\frac{3}{2}}$
(2)
(b) Simplify fully $x^{2}\left(4 x^{-\frac{1}{2}}\right)^{2}$

5*. (a) Find the value of $16^{-\frac{1}{4}}$
(b) Simplify $x\left(2 x^{-\frac{1}{4}}\right)^{4}$

6*. (a) Evaluate (32) ${ }^{5^{\frac{3}{3}}}$, giving your answer as an integer.
(b) Simplify fully $\left(\frac{25 x^{4}}{4}\right)^{-2}$
(Total 4 marks)

7*. (a) Find the value of $8^{\frac{5}{3}}$
(2)
(b) Simplify fully $\frac{\left(2 x^{\frac{1}{2}}\right)^{3}}{4 x^{2}}$
(Total 5 marks)

8*. Express $8^{2 x+3}$ in the form $2^{y}$, stating $y$ in terms of $x$.
(Total 2 marks)
$\qquad$

9*. Express $9^{3 x+1}$ in the form $3^{y}$, giving $y$ in the form $a x+b$, where $a$ and $b$ are constants.
$\qquad$

10*. $\quad \mathrm{f}(x)=x^{2}-8 x+19$
(a) Express $\mathrm{f}(x)$ in the form $(x+a)^{2}+b$, where $a$ and $b$ are constants.

The curve $C$ with equation $y=\mathrm{f}(x)$ crosses the $y$-axis at the point $P$ and has a minimum point at the point $Q$.
(b) Sketch the graph of $C$ showing the coordinates of point $P$ and the coordinates of point $Q$.
(c) Find the distance $P Q$, writing your answer as a simplified surd.

11*. $\mathrm{f}(x)=x^{2}+(k+3) x+k$,
where $k$ is a real constant.
(a) Find the discriminant of $\mathrm{f}(x)$ in terms of $k$.
(b) Show that the discriminant of $\mathrm{f}(x)$ can be expressed in the form $(k+a)^{2}+b$, where $a$ and $b$ are integers to be found.
(c) Show that, for all values of $k$, the equation $\mathrm{f}(x)=0$ has real roots.
where $p$ and $q$ are integers.
(a) Find the value of $p$ and the value of $q$.
(b) Calculate the discriminant of $4 x-5-x^{2}$.
(c) Sketch the curve with equation $y=4 x-5-x^{2}$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.
(3)
(Total 8 marks)

13*. Given that $y=2^{x}$,
(a) express $4^{x}$ in terms of $y$.
(b) Hence, or otherwise, solve

$$
8\left(4^{x}\right)-9\left(2^{x}\right)+1=0 .
$$

14*. Factorise completely $x-4 x^{3}$
(Total 3 marks)

15*. Factorise fully $25 x-9 x^{3}$
(Total 3 marks)
16. $\mathrm{f}(x)=2 x^{3}-7 x^{2}-10 x+24$.
(a) Use the factor theorem to show that $(x+2)$ is a factor of $\mathrm{f}(x)$.
(b) Factorise $\mathrm{f}(x)$ completely.
17. $\mathrm{f}(x)=2 x^{3}-7 x^{2}+4 x+4$.
(a) Use the factor theorem to show that $(x-2)$ is a factor of $\mathrm{f}(x)$.
(b) Factorise $\mathrm{f}(x)$ completely.

## Write your name here



## AS and A level Mathematics <br> Practice Paper Pure Mathematics - Algebra (part 2)



## You must have: <br> Mathematical Formulae and Statistical Tables (Pink)

Total Marks

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.


## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 12 questions in this question paper. The total mark for this paper is 85 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.
- Calculators must not be used for all questions.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Solve the simultaneous equations

$$
\begin{gathered}
x+y=2 \\
4 y^{2}-x^{2}=11
\end{gathered}
$$

2. Solve the simultaneous equations

$$
\begin{aligned}
& y+4 x+1=0 \\
& y^{2}+5 x^{2}+2 x=0
\end{aligned}
$$

(Total 6 marks)
3. Solve the simultaneous equations

$$
\begin{array}{r}
y-2 x-4=0 \\
4 x^{2}+y^{2}+20 x=0
\end{array}
$$

4. Given the simultaneous equations

$$
\begin{array}{r}
2 x+y=1 \\
x^{2}-4 k y+5 k=0
\end{array}
$$

where $k$ is a non zero constant,
(a) show that $x^{2}+8 k x+k=0$.

Given that $x^{2}+8 k x+k=0$ has equal roots,
(b) find the value of $k$.
(c) For this value of $k$, find the solution of the simultaneous equations.
5. Find the set of values of $x$ for which
(a) $4 x-5>15-x$,
(b) $x(x-4)>12$.
6. Find the set of values of $x$ for which
(a) $2(3 x+4)>1-x$,
(b) $3 x^{2}+8 x-3<0$.
7. Find the set of values of $x$ for which
(a) $3 x-7>3-x$,
(b) $x^{2}-9 x \leq 36$,
(c) both $3 x-7>3-x$ and $x^{2}-9 x \leq 36$.
8. The equation $x^{2}+(k-3) x+(3-2 k)=0$, where $k$ is a constant, has two distinct real roots.
(a) Show that $k$ satisfies

$$
k^{2}+2 k-3>0
$$

(b) Find the set of possible values of $k$.
9. The equation

$$
(k+3) x^{2}+6 x+k=5, \text { where } k \text { is a constant, }
$$

has two distinct real solutions for $x$.
(a) Show that $k$ satisfies

$$
k^{2}-2 k-24<0
$$

(b) Hence find the set of possible values of $k$.
10. The equation

$$
(p-1) x^{2}+4 x+(p-5)=0, \text { where } p \text { is a constant, }
$$

has no real roots.
(a) Show that $p$ satisfies $p^{2}-6 p+1>0$.
(b) Hence find the set of possible values of $p$.
11. The straight line with equation $y=3 x-7$ does not cross or touch the curve with equation $y=2 p x^{2}-6 p x+4 p$, where $p$ is a constant.
(a) Show that $4 p^{2}-20 p+9<0$.
(b) Hence find the set of possible values of $p$.
12.


## Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.
The six edges are straight lines.
The lengths shown in the diagram are given in metres.
Given that the perimeter of the garden is greater than 40 m ,
(a) show that $x>1.7$.

Given that the area of the garden is less than $120 \mathrm{~m}^{2}$,
(b) form and solve a quadratic inequality in $x$.
(c) Hence state the range of the possible values of $x$.

## Write your name here



## AS and $A$ level Mathematics

## Practice Paper

Pure Mathematics - Binomial expansion


## You must have: <br> Mathematical Formulae and Statistical Tables (Pink)

Total Marks

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.


## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 49 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of

$$
(2-3 x)^{5}
$$

giving each term in its simplest form.
2. Find the first 4 terms, in ascending powers of $x$, of the binomial expansion of

$$
\left(\begin{array}{c}
1 \therefore \check{C}^{5} \\
3-\frac{-x}{3}
\end{array}\right.
$$

giving each term in its simplest form.
(Total 4 marks)
3. Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of

$$
\left(2-\frac{x}{4}\right)^{10}
$$

giving each term in its simplest form.
(Total 4 marks)
4. Find the first 4 terms, in ascending powers of $x$, of the binomial expansion of

$$
\left(1+\frac{3 x}{2}\right)^{8}
$$

giving each term in its simplest form.
5. (a) Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of

$$
(3+b x)^{5}
$$

where $b$ is a non-zero constant. Give each term in its simplest form.

Given that, in this expansion, the coefficient of $x^{2}$ is twice the coefficient of $x$,
(b) find the value of $b$.
6. (a) Find the first 4 terms of the binomial expansion, in ascending powers of $x$, of

$$
\left(1+\frac{x}{4}\right)^{8}
$$

giving each term in its simplest form.
(b) Use your expansion to estimate the value of $(1.025)^{8}$, giving your answer to 4 decimal places.
7. (a) Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of $(2-3 x)^{6}$, giving each term in its simplest form.
(b) Hence, or otherwise, find the first 3 terms, in ascending powers of $x$, of the expansion of

$$
\left(\begin{array}{c}
\left.1+\begin{array}{c}
x \\
\frac{2}{2}
\end{array}\right)(2-3 x)^{6} .
\end{array}\right.
$$

8. Given that $\binom{40}{4}=40!4!$,
(a) write down the value of $b$.

In the binomial expansion of $(1+x)^{40}$, the coefficients of $x^{4}$ and $x^{5}$ are $p$ and $q$ respectively.
(b) Find the value of $\frac{q}{p}$.
9. (a) Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of

$$
(2-9 x)^{4},
$$

giving each term in its simplest form.

$$
\begin{equation*}
\mathrm{f}(x)=(1+k x)(2-9 x)^{4}, \quad \text { where } k \text { is a constant. } \tag{4}
\end{equation*}
$$

The expansion, in ascending powers of $x$, of $\mathrm{f}(x)$ up to and including the term in $x^{2}$ is

$$
A-232 x+B x^{2}
$$

where $A$ and $B$ are constants.
(b) Write down the value of $A$.
(c) Find the value of $k$.
(d) Hence find the value of $B$.

## Write your name here



Pearson Edexcel GCE


## AS and A level Mathematics

Practice Paper
Pure Mathematics - Coordinate geometry


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You must have:
Mathematical Formulae and Statistical Tables (Pink)

\section*{Instructions}
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 12 questions in this question paper. The total mark for this paper is 100 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1*. The points \(P\) and \(Q\) have coordinates \((-1,6)\) and \((9,0)\) respectively.
The line \(l\) is perpendicular to \(P Q\) and passes through the mid-point of \(P Q\).
Find an equation for \(l\), giving your answer in the form \(a x+b y+c=0\), where \(a, b\) and \(c\) are integers.
(Total 5 marks)

2*.


Figure 1
Figure 1 shows a right angled triangle \(L M N\).
The points \(L\) and \(M\) have coordinates \((-1,2)\) and \((7,-4)\) respectively.
(a) Find an equation for the straight line passing through the points \(L\) and \(M\).

Give your answer in the form \(a x+b y+c=0\), where \(a, b\) and \(c\) are integers.

Given that the coordinates of point \(N\) are \((16, p)\), where \(p\) is a constant, and angle \(L M N=90^{\circ}\),
(b) find the value of \(p\).

Given that there is a point \(K\) such that the points \(L, M, N\), and \(K\) form a rectangle,
(c) find the \(y\) coordinate of \(K\).

3*.


Not to scale

Figure 1
The straight line \(l_{1}\), shown in Figure 1, has equation \(5 y=4 x+10\)
The point \(P\) with \(x\) coordinate 5 lies on \(l_{1}\)
The straight line \(l_{2}\) is perpendicular to \(l_{1}\) and passes through \(P\).
(a) Find an equation for \(l_{2}\), writing your answer in the form \(a x+b y+c=0\) where \(a, b\) and \(c\) are integers.

The lines \(l_{1}\) and \(l_{2}\) cut the \(x\)-axis at the points \(S\) and \(T\) respectively, as shown in Figure 1.
(b) Calculate the area of triangle SPT.

4*.


Figure 2

The line \(l_{1}\), shown in Figure 2 has equation \(2 x+3 y=26\).
The line \(l_{2}\) passes through the origin \(O\) and is perpendicular to \(l_{1}\).
(a) Find an equation for the line \(l_{2}\).

The line \(l_{2}\) intersects the line \(l_{1}\) at the point \(C\). Line \(l_{1}\) crosses the \(y\)-axis at the point \(B\) as shown in Figure 2.
(b) Find the area of triangle \(O B C\). Give your answer in the form \(\frac{a}{b}\), where \(a\) and \(b\) are integers to be determined.

5*. The line \(L_{1}\) has equation \(4 y+3=2 x\).
The point \(A(p, 4)\) lies on \(L_{1}\).
(a) Find the value of the constant \(p\).

The line \(L_{2}\) passes through the point \(C(2,4)\) and is perpendicular to \(L_{1}\).
(b) Find an equation for \(L_{2}\) giving your answer in the form \(a x+b y+c=0\), where \(a, b\) and \(c\) are integers.

The line \(L_{1}\) and the line \(L_{2}\) intersect at the point \(D\).
(c) Find the coordinates of the point \(D\).
(d) Show that the length of \(C D\) is \(\frac{3}{2} \sqrt{ }\).

A point \(B\) lies on \(L_{1}\) and the length of \(A B=\sqrt{ } 80\).
The point \(E\) lies on \(L_{2}\) such that the length of the line \(C D E=3\) times the length of \(C D\).
(e) Find the area of the quadrilateral \(A C B E\).

6*


Figure 3
The points \(P(0,2)\) and \(Q(3,7)\) lie on the line \(l_{1}\), as shown in Figure 3 .
The line \(l_{2}\) is perpendicular to \(l_{1}\), passes through \(Q\) and crosses the \(x\)-axis at the point \(R\), as shown in Figure 3.

Find
(a) an equation for \(l_{2}\), giving your answer in the form \(a x+b y+c=0\), where \(a, b\) and \(c\) are integers,
(b) the exact coordinates of \(R\),
(c) the exact area of the quadrilateral \(O R Q P\), where \(O\) is the origin.
(Total 12 marks)
7. A circle \(C\) has centre \((-1,7)\) and passes through the point \((0,0)\). Find an equation for \(C\).
(Total 4 marks)
8.


Figure 4
The circle \(C\) has centre \(P(7,8)\) and passes through the point \(Q(10,13)\), as shown in Figure 4.
(a) Find the length \(P Q\), giving your answer as an exact value.
(b) Hence write down an equation for \(C\).

The line \(l\) is a tangent to \(C\) at the point \(Q\), as shown in Figure 4 .
(c) Find an equation for \(l\), giving your answer in the form \(a x+b y+c=0\), where \(a, b\) and \(c\) are integers.
9. The circle \(C\) has equation
\[
x^{2}+y^{2}+4 x-2 y-11=0 .
\]

Find
(a) the coordinates of the centre of \(C\),
(b) the radius of \(C\),
(c) the coordinates of the points where \(C\) crosses the \(y\)-axis, giving your answers as simplified surds.
10. The circle \(C\) has equation
\[
x^{2}+y^{2}-10 x+6 y+30=0
\]

Find
(a) the coordinates of the centre of \(C\),
(b) the radius of \(C\),
(c) the \(y\) coordinates of the points where the circle \(C\) crosses the line with equation \(x=4\), giving your answers as simplified surds.
(Total 7 marks)
11.


Figure 5
Figure 5 shows a circle \(C\) with centre \(Q\) and radius 4 and the point \(T\) which lies on \(C\). The tangent to \(C\) at the point \(T\) passes through the origin \(O\) and \(O T=6 \sqrt{ } 5\).

Given that the coordinates of \(Q\) are \((11, k)\), where \(k\) is a positive constant,
(a) find the exact value of \(k\),
(b) find an equation for \(C\).
12. The circle \(C\), with centre \(A\), passes through the point \(P\) with coordinates \((-9,8)\) and the point \(Q\) with coordinates \((15,-10)\).

Given that \(P Q\) is a diameter of the circle \(C\),
(a) find the coordinates of \(A\),
(b) find an equation for \(C\).

A point \(R\) also lies on the circle \(C\).
Given that the length of the chord \(P R\) is 20 units,
(c) find the length of the shortest distance from \(A\) to the chord \(P R\).

Give your answer as a surd in its simplest form.
(d) Find the size of the angle \(A R Q\), giving your answer to the nearest 0.1 of a degree.

Write your name here


\section*{AS and \(A\) level Mathematics \\ Practice Paper \\ Pure Mathematics - Differentiation 1}


\section*{You must have: \\ Mathematical Formulae and Statistical Tables (Pink)}

Total Marks

\section*{Instructions}
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 99 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1*. Given
\[
y=\sqrt{x}+\frac{4}{\sqrt{x}}+4, \quad x>0
\]
find the value of \(\frac{\mathrm{d} y}{\mathrm{~d} x}\) when \(x=8\), writing your answer in the form \(a \sqrt{2}\), where \(a\) is a rational number.
(Total 5 marks)

2*. Given that
\[
y=3 x^{2}+6 x^{\frac{1}{3}}+\frac{2 x^{3}-7}{3 \sqrt{x}}, \quad x>0,
\]
find \(\frac{\mathrm{d} y}{\mathrm{~d} x}\). Give each term in your answer in its simplified form.
(Total 6 marks)

3*. Differentiate with respect to \(x\), giving answer in its simplest form
\[
\frac{x^{5}+6 \sqrt{x}}{2 x^{2}}
\]

4*.
\[
y=5 x^{3}-6 x^{\frac{4}{3}}+2 x-3
\]
(a) Find \(\frac{\mathrm{d} y}{\mathrm{~d} x}\), giving each term in its simplest form.
(b) Find \(\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}\)


\section*{Figure 1}

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius \(x \mathrm{~mm}\) and height hmm , as shown in Figure 1.

Given that the volume of each tablet has to be \(60 \mathrm{~mm}^{3}\),
(a) express \(h\) in terms of \(x\),
(b) show that the surface area, \(A \mathrm{~mm}^{2}\), of a tablet is given by \(\mathrm{A}=2 \pi x^{2}+\frac{120}{x}\).

The manufacturer needs to minimise the surface area \(A \mathrm{~mm}^{2}\), of a tablet.
(c) Use calculus to find the value of \(x\) for which \(A\) is a minimum.
(d) Calculate the minimum value of \(A\), giving your answer to the nearest integer.
(e) Show that this value of \(A\) is a minimum.
6. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of \(75 \pi \mathrm{~cm}^{3}\).

The cost of polishing the surface area of this glass cylinder is \(£ 2\) per \(\mathrm{cm}^{2}\) for the curved surface area and \(£ 3 \mathrm{per} \mathrm{cm}^{2}\) for the circular top and base areas.

Given that the radius of the cylinder is \(r \mathrm{~cm}\),
(a) show that the cost of the polishing, \(£ C\), is given by
\[
C=6 \pi r^{2}+\frac{300 \pi}{r} .
\]
(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.
(c) Justify that the answer that you have obtained in part (b) is a minimum.


\section*{Figure 2}

Figure 2 shows a flowerbed. Its shape is a quarter of a circle of radius \(x\) metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to \(x\) metres and width equal to \(y\) metres.

Given that the area of the flowerbed is \(4 \mathrm{~m}^{2}\),
(a) show that
\[
y=\frac{16-\pi x^{2}}{8 x} .
\]
(b) Hence show that the perimeter \(P\) metres of the flowerbed is given by the equation
\[
P=\frac{8}{x}+2 x .
\]
(c) Use calculus to find the minimum value of \(P\).
(d) Find the width of each rectangle when the perimeter is a minimum.

Give your answer to the nearest centimetre.
8.


Figure 3

Figure 3 shows the plan of a pool.
The shape of the pool \(A B C D E F A\) consists of a rectangle \(B C E F\) joined to an equilateral triangle \(B F A\) and a semi-circle \(C D E\), as shown in Figure 3.
Given that \(A B=x\) metres, \(E F=y\) metres, and the area of the pool is \(50 \mathrm{~m}^{2}\),
(a) show that
\[
y=\frac{50}{x}-\frac{x}{8}(\pi+2 \sqrt{ } 3)
\]
(b) Hence show that the perimeter, \(P\) metres, of the pool is given by
\[
P=\frac{100}{x}+\frac{x}{4}(\pi+8-2 \sqrt{ } 3)
\]
(c) Use calculus to find the minimum value of \(P\), giving your answer to 3 significant figures.
(d) Justify, by further differentiation, that the value of \(P\) that you have found is a minimum.
9. The volume \(V \mathrm{~cm}^{3}\) of a box, of height \(x \mathrm{~cm}\), is given by
\[
V=4 x(5-x)^{2}, \quad 0<x<5 .
\]
(a) Find \(\frac{\mathrm{d} V}{\mathrm{~d} x}\).
(b) Hence find the maximum volume of the box.
(c) Use calculus to justify that the volume that you found in part (b) is a maximum.
10. Joan brings a cup of hot tea into a room and places the cup on a table. At time \(t\) minutes after Joan places the cup on the table, the temperature, \(\theta^{\circ} \mathrm{C}\), of the tea is modelled by the equation
\[
\theta=20+A \mathrm{e}^{-k t}
\]
where \(A\) and \(k\) are positive constants.
Given that the initial temperature of the tea was \(90^{\circ} \mathrm{C}\),
(a) find the value of \(A\).

The tea takes 5 minutes to decrease in temperature from \(90^{\circ} \mathrm{C}\) to \(55^{\circ} \mathrm{C}\).
(b) Show that \(k=\frac{1}{5} \ln 2\).
(c) Find the rate at which the temperature of the tea is decreasing at the instant when \(t=10\). Give your answer, in \({ }^{\circ} \mathrm{C}\) per minute, to 3 decimal places.
11. The mass, \(m\) grams, of a leaf \(t\) days after it has been picked from a tree is given by
\[
m=p \mathrm{e}^{-k t}
\]
where \(k\) and \(p\) are positive constants.
When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.
(a) Write down the value of \(p\).
(b) Show that \(k=\frac{1}{4} \ln 3\).
(c) Find the value of \(t\) when \(\frac{\mathrm{d} t}{\mathrm{~d} t}=-0.6 \ln 3\).

Write your name here


\section*{AS and \(A\) level Mathematics \\ Practice Paper \\ Pure Mathematics - Differentiation 2}


\section*{You must have: \\ Mathematical Formulae and Statistical Tables (Pink)}

Total Marks

\section*{Instructions}
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 100 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1*. The curve \(C_{1}\) has equation
\[
y=x^{2}(x+2) .
\]
(a) Find \(\frac{\mathrm{d} y}{\mathrm{~d} x}\).
(b) Sketch \(C_{1}\), showing the coordinates of the points where \(C_{1}\) meets the \(x\)-axis.
(c) Find the gradient of \(C_{1}\) at each point where \(C_{1}\) meets the \(x\)-axis.

The curve \(C_{2}\) has equation
\[
y=(x-k)^{2}(x-k+2),
\]
where \(k\) is a constant and \(k>2\).
(d) Sketch \(C_{2}\), showing the coordinates of the points where \(C_{2}\) meets the \(x\) and \(y\) axes.

2*. The curve \(C\) has equation
\[
y=(x+1)(x+3)^{2} .
\]
(a) Sketch \(C\), showing the coordinates of the points at which \(C\) meets the axes.
(b) Show that \(\frac{\mathrm{d} y}{\mathrm{~d} x}=3 x^{2}+14 x+15\).

The point \(A\), with \(x\)-coordinate -5 , lies on \(C\).
(c) Find the equation of the tangent to \(C\) at \(A\), giving your answer in the form \(y=m x+c\), where \(m\) and \(c\) are constants.

Another point \(B\) also lies on \(C\). The tangents to \(C\) at \(A\) and \(B\) are parallel.
(d) Find the \(x\)-coordinate of \(B\).
\(3^{*}\). The curve \(C\) has equation
\[
y=\frac{\left(x^{2}+4\right)(x-3)}{2 x}, x \neq 0 .
\]
(a) Find \(\frac{\mathrm{d} y}{\mathrm{~d} x}\) in its simplest form.
(b) Find an equation of the tangent to \(C\) at the point where \(x=-1\).

Give your answer in the form \(a x+b y+c=0\), where \(a, b\) and \(c\) are integers.

4*. The curve \(C\) has equation \(y=2 x^{3}+k x^{2}+5 x+6\), where \(k\) is a constant.
(a) Find \(\frac{\mathrm{d} y}{\mathrm{~d} x}\).

The point \(P\), where \(x=-2\), lies on \(C\).
The tangent to \(C\) at the point \(P\) is parallel to the line with equation \(2 y-17 x-1=0\).
Find
(b) the value of \(k\),
(c) the value of the \(y\) coordinate of \(P\),
(d) the equation of the tangent to \(C\) at \(P\), giving your answer in the form \(a x+b y+c=0\), where \(a, b\) and \(c\) are integers.
\(5^{*}\). The curve \(C\) has equation
\[
y=2 x-8 \sqrt{ } x+5, \quad x \geq 0
\]
(a) Find \(\frac{\mathrm{d} y}{\mathrm{~d} x}\), giving each term in its simplest form.

The point \(P\) on \(C\) has \(x\)-coordinate equal to \(\frac{1}{4}\).
(b) Find the equation of the tangent to \(C\) at the point \(P\), giving your answer in the form \(y=a x+b\), where \(a\) and \(b\) are constants.

The tangent to \(C\) at the point \(Q\) is parallel to the line with equation \(2 x-3 y+18=0\).
(c) Find the coordinates of \(Q\).

6*. The curve \(C\) has equation
\[
y=\frac{1}{2} x^{3}-9 x^{\frac{3}{2}}+\frac{8}{x}+30, \quad x>0 .
\]
(a) Find \(\frac{\mathrm{d} y}{\mathrm{~d} x}\)
(b) Show that the point \(P(4,-8)\) lies on \(C\)
(c) Find an equation of the normal to \(C\) at the point \(P\), giving your answer in the form \(a x+b y+c=0\), where \(\mathrm{a}, \mathrm{b}\) and c are integers.


Figure 1

Figure 1 shows a sketch of the curve \(C\) with equation
\[
y=2-\frac{1}{x}, \quad x \neq 0 .
\]

The curve crosses the \(x\)-axis at the point \(A\).
(a) Find the coordinates of \(A\).
(b) Show that the equation of the normal to \(C\) at \(A\) can be written as
\[
2 x+8 y-1=0 .
\]

The normal to \(C\) at \(A\) meets \(C\) again at the point \(B\), as shown in Figure 1 .
(c) Find the coordinates of \(B\).

8*.


Figure 2

A sketch of part of the curve \(C\) with equation
\[
y=20-4 x-\frac{18}{x}, \quad x>0
\]
is shown in Figure 2.
Point \(A\) lies on \(C\) and has an \(x\) coordinate equal to 2 .
(a) Show that the equation of the normal to \(C\) at \(A\) is \(y=-2 x+7\).

The normal to \(C\) at \(A\) meets \(C\) again at the point \(B\), as shown in Figure 2.
(b) Use algebra to find the coordinates of \(B\).
9.


Figure 3

Figure 3 shows a sketch of part of the curve with equation
\[
y=4 x^{3}+9 x^{2}-30 x-8, \quad-0.5 \leqslant x \leqslant 2.2
\]

The curve has a turning point at the point \(A\).
(a) Using calculus, show that the \(x\) coordinate of \(A\) is 1

The curve crosses the \(x\)-axis at the points \(B(2,0)\) and \(C\left({ }_{-}{ }^{1}, 0\right.\) : \(\overline{4} \div\)

The finite region \(R\), shown shaded in Figure 3, is bounded by the curve, the line \(A B\), and the \(x\)-axis.
(b) Use integration to find the area of the finite region \(R\), giving your answer to 2 decimal places.

\section*{Write your name here}


\section*{AS and A level Mathematics}

\section*{Practice Paper}

Pure Mathematics - Exponentials and logarithms

\section*{You must have: \\ Mathematical Formulae and Statistical Tables (Pink)}

Total Marks


\section*{Instructions}
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 12 questions in this question paper. The total mark for this paper is 79 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
1. Find the values of \(x\) such that
\[
2 \log _{3} x-\log _{3}(x-2)=2
\]
2. Given that \(y=3 x^{2}\),
(a) show that \(\log _{3} y=1+2 \log _{3} x\).
(b) Hence, or otherwise, solve the equation
\[
1+2 \log _{3} x=\log _{3}(28 x-9)
\]
3. Given that \(2 \log _{2}(x+15)-\log _{2} x=6\),
(a) show that \(x^{2}-34 x+225=0\).
(b) Hence, or otherwise, solve the equation \(2 \log _{2}(x+15)-\log _{2} x=6\).
4. (i) Find the exact value of \(x\) for which
\[
\log _{2}(2 x)=\log _{2}(5 x+4)-3 .
\]
(ii) Given that
\[
\log _{a} y+3 \log _{a} 2=5
\]
express \(y\) in terms of \(a\).
Give your answer in its simplest form.
5. (i) \(2 \log (x+a)=\log \left(16 a^{6}\right)\), where \(a\) is a positive constant Find \(x\) in terms of \(a\), giving your answer in its simplest form.
(ii) \(\quad \log _{3}(9 y+b)-\log _{3}(2 y-b)=2\), where \(b\) is a positive constant Find \(y\) in terms of \(b\), giving your answer in its simplest form.
6. Find, giving your answer to 3 significant figures where appropriate, the value of \(x\) for which
(a) \(5^{x}=10\),
(b) \(\log _{3}(x-2)=-1\).
7. Find the exact solutions, in their simplest form, to the equations
(a) \(\mathrm{e}^{3 x-9}=8\)
8.
\[
\mathrm{f}(x)=-6 x^{3}-7 x^{2}+40 x+21
\]
(a) Use the factor theorem to show that \((x+3)\) is a factor of \(\mathrm{f}(x)\)
(b) Factorise \(\mathrm{f}(x)\) completely.
(c) Hence solve the equation
\[
6\left(2^{3 y}\right)+7\left(2^{2 y}\right)=40\left(2^{y}\right)+21
\]
giving your answer to 2 decimal places.
9. (i) Use logarithms to solve the equation \(8^{2 x+1}=24\), giving your answer to 3 decimal places.
(ii) Find the values of \(y\) such that
\[
\log _{2}(11 y-3)-\log _{2} 3-2 \log _{2} y=1, \quad y>\frac{3}{11}
\]
10. (i) Given that
\[
\log _{3}(3 b+1)-\log _{3}(a-2)=-1, \quad a>2,
\]
express \(b\) in terms of \(a\).
(ii) Solve the equation
\[
2^{2 x+5}-7\left(2^{x}\right)=0,
\]
giving your answer to 2 decimal places.
(Solutions based entirely on graphical or numerical methods are not acceptable.)
11. (i) Solve
\[
5^{y}=8
\]
giving your answers to 3 significant figures.
(ii) Use algebra to find the values of \(x\) for which
\[
\log _{2}(x+15)-4=\frac{1}{2} \log _{2} x
\]
12. (a) Sketch the graph of
\[
y=3^{x}, x \in \mathbb{R},
\]
showing the coordinates of any points at which the graph crosses the axes.
(b) Use algebra to solve the equation \(3^{2 x}-9\left(3^{x}\right)+18=0\), giving your answers to 2 decimal places where appropriate.

\section*{Write your name here}


\section*{AS and A level Mathematics}

\section*{Practice Paper}

Pure Mathematics - Graphs and transformations

\section*{You must have: \\ Mathematical Formulae and Statistical Tables (Pink)}

Total Marks

\section*{Instructions}
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 12 questions in this question paper. The total mark for this paper is 100 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.
- Calculators must not be used for all questions.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
1.


\section*{Figure 1}

Figure 1 shows a sketch of the curve \(C\) with equation
\[
y=\frac{1}{x}+1, \quad x \neq 0 .
\]

The curve \(C\) crosses the \(x\)-axis at the point \(A\).
(a) State the \(x\)-coordinate of the point \(A\).

The curve \(D\) has equation \(y=x^{2}(x-2)\), for all real values of \(x\).
(b) On a copy of Figure 1, sketch a graph of curve \(D\). Show the coordinates of each point where the curve \(D\) crosses the coordinate axes.
(c) Using your sketch, state, giving a reason, the number of real solutions to the equation
\[
x^{2}(x-2)=\frac{1}{x}+1
\]
2.


Figure 2
Figure 2 shows a sketch of part of the curve with equation \(y=\mathrm{f}(x)\). The curve has a maximum point \(A\) at \((-2,4)\) and a minimum point \(B\) at \((3,-8)\) and passes through the origin \(O\).

On separate diagrams, sketch the curve with equation
(a) \(y=3 \mathrm{f}(x)\),
(b) \(y=\mathrm{f}(x)-4\).

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the \(y\)-axis.
(Total 5 marks)
3.


Figure 3

Figure 3 shows a sketch of the curve with equation \(y=\mathrm{f}(x)\) where
\[
\mathrm{f}(x)=\frac{x}{x-2}, \quad x \neq 2
\]

The curve passes through the origin and has two asymptotes, with equations \(y=1\) and \(x=2\), as shown in Figure 1.
(a) In the space below, sketch the curve with equation \(y=\mathrm{f}(x-1)\) and state the equations of the asymptotes of this curve.
(b) Find the coordinates of the points where the curve with equation \(y=\mathrm{f}(x-1)\) crosses the coordinate axes.
4.


Figure 4

Figure 4 shows a sketch of the curve with equation \(y=\frac{2}{x}, x \neq 0\).
The curve \(C\) has equation \(y=\frac{2}{x}-5, x \neq 0\), and the line \(l\) has equation \(y=4 x+2\).
(a) Sketch and clearly label the graphs of \(C\) and \(l\) on a single diagram.

On your diagram, show clearly the coordinates of the points where \(C\) and \(l\) cross the coordinate axes.
(b) Write down the equations of the asymptotes of the curve \(C\).
(c) Find the coordinates of the points of intersection of \(y=\frac{2}{x}-5\) and \(y=4 x+2\).
5. (a) Factorise completely \(9 x-4 x^{3}\).
(b) Sketch the curve \(C\) with equation
\[
y=9 x-4 x^{3}
\]

Show on your sketch the coordinates at which the curve meets the \(x\)-axis.

The points \(A\) and \(B\) lie on \(C\) and have \(x\) coordinates of -2 and 1 respectively.
(c) Show that the length of \(A B\) is \(k \sqrt{ } 10\), where \(k\) is a constant to be found.
6.


Figure 5
Figure 5 shows a sketch of the curve \(C\) with equation \(y=\mathrm{f}(x)\).
The curve \(C\) passes through the origin and through \((6,0)\).
The curve \(C\) has a minimum at the point \((3,-1)\).
On separate diagrams, sketch the curve with equation
(a) \(y=\mathrm{f}(2 x)\),
(b) \(y=-\mathrm{f}(x)\),
(c) \(y=\mathrm{f}(x+p)\), where \(p\) is a constant and \(0<p<3\).

On each diagram show the coordinates of any points where the curve intersects the \(x\)-axis and of any minimum or maximum points.
7.


Figure 6

Figure 6 shows a sketch of the curve with equation \(y=\mathrm{f}(x)\) where
\[
\mathrm{f}(x)=(x+3)^{2}(x-1), \quad x \in \mathbb{R} .
\]

The curve crosses the \(x\)-axis at \((1,0)\), touches it at \((-3,0)\) and crosses the \(y\)-axis at \((0,-9)\).
(a) Sketch the curve \(C\) with equation \(y=\mathrm{f}(x+2)\) and state the coordinates of the points where the curve \(C\) meets the \(x\)-axis.
(b) Write down an equation of the curve \(C\).
(c) Use your answer to part (b) to find the coordinates of the point where the curve \(C\) meets the \(y\)-axis.
(Total 6 marks)
8. (a) On separate axes sketch the graphs of
(i) \(y=-3 x+c\), where \(c\) is a positive constant,
(ii) \(y=\frac{1}{x}+5\)

On each sketch show the coordinates of any point at which the graph crosses the \(y\)-axis and the equation of any horizontal asymptote.

Given that \(y=-3 x+c\), where \(c\) is a positive constant, meets the curve \(y=\frac{1}{x}+5\) at two
distinct points,
(b) show that \((5-c)^{2}>12\)
(c) Hence find the range of possible values for \(c\).
9.
\[
4 x^{2}+8 x+3 \equiv a(x+b)^{2}+c
\]
(a) Find the values of the constants \(a, b\) and \(c\).
(b) Sketch the curve with equation \(y=4 x^{2}+8 x+3\), showing clearly the coordinates of any points where the curve crosses the coordinate axes.
(Total 7 marks)
10.


Figure 7

Figure 7 shows a sketch of the curve \(C\) with equation \(y=\mathrm{f}(x)\), where
\[
\mathrm{f}(x)=x^{2}(9-2 x) .
\]

There is a minimum at the origin, a maximum at the point \((3,27)\) and \(C\) cuts the \(x\)-axis at the point \(A\).
(a) Write down the coordinates of the point \(A\).
(b) On separate diagrams sketch the curve with equation
(i) \(y=\mathrm{f}(x+3)\),
(ii) \(y=\mathrm{f}(3 x)\).

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

The curve with equation \(y=\mathrm{f}(x)+k\), where \(k\) is a constant, has a maximum point at \((3,10)\).
(c) Write down the value of \(k\).
11.


Figure 8
Figure 8 shows a sketch of part of the curve \(y=\mathrm{f}(x), x \in \mathbb{R}\), where
\[
\mathrm{f}(x)=(2 x-5)^{2}(x+3)
\]
(a) Given that
(i) the curve with equation \(y=\mathrm{f}(x)-k, x \in \mathbb{R}\), passes through the origin, find the value of the constant \(k\),
(ii) the curve with equation \(y=\mathrm{f}(x+c), x \in \mathbb{R}\), has a minimum point at the origin, find the value of the constant \(c\).
(b) Show that \(\mathrm{f}^{\prime}(x)=12 x^{2}-16 x-35\)

Points \(A\) and \(B\) are distinct points that lie on the curve \(y=\mathrm{f}(x)\).
The gradient of the curve at \(A\) is equal to the gradient of the curve at \(B\).
Given that point \(A\) has \(x\) coordinate 3
(c) find the \(x\) coordinate of point \(B\).
12. (a) Sketch the graphs of
(i) \(y=x(x+2)(3-x)\),
(ii) \(y=-\frac{2}{x}\).
showing clearly the coordinates of all the points where the curves cross the coordinate axes.
(b) Using your sketch state, giving a reason, the number of real solutions to the equation
\[
x(x+2)(3-x)+\frac{2}{x}=0 .
\]

\section*{Write your name here}


\section*{AS and \(A\) level Mathematics}

\section*{Practice Paper} Pure Mathematics - Integration

\section*{You must have: \\ Mathematical Formulae and Statistical Tables (Pink)}

Total Marks

\section*{Instructions}
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1*. Find
\[
\int\left(6 x^{2}+\frac{2}{x^{2}}+5\right) \mathrm{d} x
\]
giving each term in its simplest form.

2*. Find
\[
\int\left(2 x^{4}-\frac{4}{\sqrt{x}}+3\right) \mathrm{d} x
\]
giving each term in its simplest form.
(Total 4 marks)

3*. Find
\[
\int\left(2 x^{5}-\frac{1}{4 x^{3}}-5\right) d \mathrm{~d} x
\]
giving each term in its simplest form.
(Total 4 marks)
\(\qquad\)
4. Use integration to find
\[
\int_{1}^{\int \sqrt{3}}\left(\frac{x^{3}}{6}+\frac{1}{3 x^{2}}\right) \mathrm{d} x
\]
giving your answer in the form \(a+b \sqrt{ } 3\), where \(a\) and \(b\) are constants to be determined.
(Total 5 marks)
5.


Figure 1

Figure 1 shows a sketch of part of the curve \(C\) with equation
\[
y=\frac{1}{8} x^{3}+\frac{3}{4} x^{2}, \quad x \in \mathbb{R}
\]

The curve \(C\) has a maximum turning point at the point \(A\) and a minimum turning point at the origin \(O\).

The line \(l\) touches the curve \(C\) at the point \(A\) and cuts the curve \(C\) at the point \(B\).

The \(x\) coordinate of \(A\) is -4 and the \(x\) coordinate of \(B\) is 2 .

The finite region \(R\), shown shaded in Figure 3, is bounded by the curve \(C\) and the line \(l\).

Use integration to find the area of the finite region \(R\).

6*.
\[
\frac{\mathrm{d} y}{\mathrm{~d} x}=6 x^{-\frac{1}{2}}+x \sqrt{ } x, \quad x>0
\]

Given that \(y=37\) at \(x=4\), find \(y\) in terms of \(x\), giving each term in its simplest form.
(Total 7 marks)
\(\qquad\)

7*. A curve with equation \(y=\mathrm{f}(x)\) passes through the point \((2,10)\). Given that
\[
\mathrm{f}^{\prime}(x)=3 x^{2}-3 x+5
\]
find the value of \(f(1)\).

8*. A curve with equation \(y=\mathrm{f}(x)\) passes through the point \((4,25)\).
Given that \(\mathrm{f}^{\prime}(x)=\frac{3}{8} x^{2}-10 x^{-\frac{1}{2}}+1, \quad x>0\),
(a) find \(\mathrm{f}(x)\), simplifying each term.
(b) Find an equation of the normal to the curve at the point \((4,25)\). Give your answer in the form \(a x+b y+c=0\), where \(a, b\) and \(c\) are integers to be found.

9*. The curve \(C\) has equation \(y=\mathrm{f}(x), x>0\), where
\[
\mathrm{f}^{\prime}(x)=30+\frac{6-5 x^{2}}{\sqrt{x}}
\]

Given that the point \(P(4,-8)\) lies on \(C\),
(a) find the equation of the tangent to \(C\) at \(P\), giving your answer in the form \(y=m x+c\), where \(m\) and \(c\) are constants.
(b) Find \(\mathrm{f}(x)\), giving each term in its simplest form.

10*. A curve with equation \(y=\mathrm{f}(x)\) passes through the point \((4,9)\).
Given that
\[
\mathrm{f}^{\prime}(x)=\frac{3 \sqrt{ } x}{2}-\frac{9}{4 \sqrt{ } x}+2, x>0
\]
(a) find \(\mathrm{f}(x)\), giving each term in its simplest form.

Point \(P\) lies on the curve.
The normal to the curve at \(P\) is parallel to the line \(2 y+x=0\).
(b) Find the \(x\)-coordinate of \(P\).
11.


Figure 2

Figure 2 shows the line with equation \(y=10-x\) and the curve with equation \(y=10 x-x^{2}-8\). The line and the curve intersect at the points \(A\) and \(B\), and \(O\) is the origin.
(a) Calculate the coordinates of \(A\) and the coordinates of \(B\).

The shaded area \(R\) is bounded by the line and the curve, as shown in Figure 2.
(b) Calculate the exact area of \(R\).
12. (a) Find
\[
\int 10 x\left(x^{\frac{1}{2}}-2\right) \mathrm{d} x
\]
giving each term in its simplest form.


Figure 2

Figure 2 shows a sketch of part of the curve \(C\) with equation
\[
y=10 x\left(x^{\frac{1}{2}}-2\right), \quad x \geq 0
\]

The curve \(C\) starts at the origin and crosses the \(x\)-axis at the point \((4,0)\).
The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve \(C\), the \(x\)-axis and the line \(x=9\).
(b) Use your answer from part (a) to find the total area of the shaded regions.
13.


Figure 3

Figure 3 shows a sketch of part of the curve \(C\) with equation
\[
y=x(x+4)(x-2) .
\]

The curve \(C\) crosses the \(x\)-axis at the origin \(O\) and at the points \(A\) and \(B\).
(a) Write down the \(x\)-coordinates of the points \(A\) and \(B\).

The finite region, shown shaded in Figure 3, is bounded by the curve \(C\) and the \(x\)-axis.
(b) Use integration to find the total area of the finite region shown shaded in Figure 3.
14.


Figure 3
Figure 3 shows a sketch of part of the curve with equation
\[
y=3 x-x^{\frac{3}{2}} \quad x \geq 0 .
\]

The finite region \(S\), bounded by the \(x\)-axis and the curve, is shown shaded in Figure 3.
(a) Find
\[
\begin{equation*}
\int\left(3 x-x^{\frac{3}{2}}\right) \mathrm{d} x . \tag{3}
\end{equation*}
\]
(b) Hence find the area of \(S\).

Write your name here


\section*{AS and A level Mathematics}

\section*{Practice Paper}

Pure Mathematics - Trigonometry

\section*{You must have: \\ Mathematical Formulae and Statistical Tables (Pink)}

Total Marks

\section*{Instructions}
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 6 questions in this question paper. The total mark for this paper is 49 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
1. Solve, for \(0 \leq x<180^{\circ}\)
\[
\cos \left(3 x-10^{\circ}\right)=-0.4
\]
giving your answers to 1 decimal place. You should show each step in your working.
(Total 7 marks)
2. (a) Show that the equation
\[
\cos ^{2} x=8 \sin ^{2} x-6 \sin x
\]
can be written in the form
\[
\begin{equation*}
(3 \sin x-1)^{2}=2 \tag{3}
\end{equation*}
\]
(b) Hence solve, for \(0 \leqslant x<360^{\circ}\),
\[
\cos ^{2} x=8 \sin ^{2} x-6 \sin x
\]
giving your answers to 2 decimal places.
3. (a) Show that the equation
\[
\tan 2 x=5 \sin 2 x
\]
can be written in the form
\[
(1-5 \cos 2 x) \sin 2 x=0
\]
(b) Hence solve, for \(0 \leq x \leq 180^{\circ}\)
\[
\tan 2 x=5 \sin 2 x
\]
giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.
4. (a) Show that the equation
\[
3 \sin ^{2} x+7 \sin x=\cos ^{2} x-4
\]
can be written in the form
\[
\begin{equation*}
4 \quad \sin ^{2} x+7 \sin x+3=0 \tag{2}
\end{equation*}
\]
(b) Hence solve, for \(0 \leq x<360^{\circ}\)
\[
3 \sin ^{2} x+7 \sin x=\cos ^{2} x-4
\]
giving your answers to 1 decimal place where appropriate.
5. (i) Solve, for \(0 \leq \theta<360^{\circ}\), the equation \(9 \sin \left(\theta+60^{\circ}\right)=4\), giving your answers to 1 decimal place. You must show each step of your working.
(ii) Solve, for \(-\pi \leq x<\pi\), the equation \(2 \tan x-3 \sin x=0\), giving your answers to 2 decimal places where appropriate.
[Solutions based entirely on graphical or numerical methods are not acceptable.]
6. (i) Solve, for \(-180^{\circ} \leq x<180^{\circ}\),
\[
\tan \left(x-40^{\circ}\right)=1.5,
\]
giving your answers to 1 decimal place.
(ii) (a) Show that the equation
\[
\sin \theta \tan \theta=3 \cos \theta+2
\]
can be written in the form
\[
4 \cos ^{2} \theta+2 \cos \theta-1=0
\]
(b) Hence solve, for \(0 \leq \theta<360^{\circ}\),
\[
\sin \theta \tan \theta=3 \cos \theta+2
\]
showing each stage of your working.

\section*{Task 2 -}

\title{
Mastering \\ Applied \\ Statistics
}

Year 1

\section*{AS/A Level Mathematics}

\section*{Sampling}

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

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- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 Here are some descriptions of sampling methods.
A: The population is split into groups and a proportional representation of the different groups is selected.
B: Every member of the population has an equal chance of being selected.
C: A convenient sample is taken from any members of the population at any time.
D: A repetitive system used to select a sample from the population.
E: The population is split into groups and a certain number from each group is chosen in any order.

In the table below, match each sampling method with the letter of its description.
\begin{tabular}{|c|c|}
\hline Method & Letter \\
\hline Quota Sampling & \\
\hline Stratified Sampling & \\
\hline Simple Random Sampling & \\
\hline Opportunity Sampling & \\
\hline Systematic Sampling & \\
\hline
\end{tabular}

2 The headteacher of a school wants to find out what students think about the school. She decides to take a census rather than a sample.
(a) What is the population for the census?
(b) Give one advantage and one disadvantage of using a census.

3 A gym wants to find out what its members think about the gym's opening times and decides to carry out a survey.
(a) Suggest a suitable sampling frame for the survey
(b) Identify the sampling units

The gym decides to ask the first 20 members that come into the gym one morning to complete the survey.
(c) State the sampling technique the gym used.
(d) Give one advantage and one disadvantage of this technique.

4 Kwame wants to investigate how much time year 7 students in his school spend playing sport. He gets a list of year 7 students and selects every \(5^{\text {th }}\) student on the list to add to his sample.
(a) Write down the name of the sampling method Kwame has chosen.
(b) Give one advantage and one disadvantage of this sampling method.

5 Frank wants to find out how much time students in his school spend listening to music.
He asks students entering the school until he has asked 25 boys and 25 girls.
(a) Write down the name of the sampling method Frank has chosen.
(b) Give one advantage and one disadvantage of this sampling method.

6 The table shows information about the number of students in a school.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & \multicolumn{6}{|c|}{ Year Group } \\
Total \\
\hline & \(\mathbf{7}\) & \(\mathbf{8}\) & \(\mathbf{9}\) & \(\mathbf{1 0}\) & \(\mathbf{1 1}\) & \\
\hline Boys & 71 & 65 & 59 & 55 & 48 & 298 \\
\hline Girls & 65 & 54 & 53 & 50 & 53 & 275 \\
\hline Total & 136 & 119 & 112 & 105 & 101 & 573 \\
\hline
\end{tabular}

A teacher wants to find out what students think about the school's canteen. They decide to take a sample of 60 students stratified by age and by year group.

Calculate the number of year 11 girls in the sample.
(Total for question 6 is \(\mathbf{2}\) marks)
7 There are 550 students in a school They all study either French or German or Spanish.
\begin{tabular}{|c|c|c|c|}
\hline Language & French & Spanish & German \\
\hline \begin{tabular}{c} 
Number of \\
Students
\end{tabular} & 181 & 146 & 223 \\
\hline
\end{tabular}

An inspector decides to take a sample of 40 students stratified by the language they study.
Calculate the number of students who study German in the sample.

\title{
AS/A Level Mathematics
}

\section*{Interpolation and Standard Deviation}

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- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear.

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\section*{Information}
- The marks for each question are shown in brackets
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\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 Adam is measuring the heights in cm of his tomato plants.
\begin{tabular}{|c|c|}
\hline Height (cm) & Frequency \\
\hline \(140<\mathrm{h} \square 150\) & 7 \\
\hline \(150<\mathrm{h} \square 160\) & 10 \\
\hline \(160<\mathrm{h} \square 170\) & 15 \\
\hline \(170<\mathrm{h} \square 180\) & 19 \\
\hline \(180<\mathrm{h} \square 200\) & 9 \\
\hline
\end{tabular}
(a) Use linear interpolation to estimate the median height.
(b) Estimate the mean height.
(c) Estimate the standard deviation.

2 A company is investigating how long it takes employees, \(t\) minutes, to get to an event.
They produce a table below of coded times, \(x\) minutes, for a random sample of 50 employees.
\begin{tabular}{|c|c|}
\hline Coded Time (minutes) & Frequency \\
\hline \(0<x \square 5\) & 1 \\
\hline \(5<x \square 10\) & 9 \\
\hline \(10<x \square 15\) & 19 \\
\hline \(15<x \square 25\) & 14 \\
\hline \(25<x \square 40\) & 7 \\
\hline
\end{tabular}
(a) Use linear interpolation to estimate the median of the coded times.
(b) Estimate the standard deviation of the coded times.

The coded data was calculated sing the formula: \(x=\frac{t-20}{2}\)
(c) Calculate the median and the standard deviation of \(t\).

3 The distance travelled by 100 people to an event is summarised below.
\begin{tabular}{|c|c|}
\hline Distance (nearest mile) & Frequency \\
\hline \(0-9\) & 4 \\
\hline \(10-19\) & 19 \\
\hline \(20-29\) & 41 \\
\hline \(30-39\) & 26 \\
\hline \(40-49\) & 9 \\
\hline \(50-59\) & 1 \\
\hline
\end{tabular}

You may use: \(\quad \Sigma f x=2651 \quad \Sigma f x^{2}=80434.25\)
(a) Use linear interpolation to estimate the median height.
(b) Estimate the mean height.
(c) Estimate the standard deviation.

4 The times 12 athletes took to run 400 m is summarised in the table below.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Athlete & A & B & C & D & E & F & G & H & A & I & J & K \\
\hline Time (s) & 45.2 & 46.9 & 46.1 & 46.2 & 45.4 & 45.1 & 47.8 & 45.4 & 45.5 & 46.1 & 46.4 & 45.7 \\
\hline
\end{tabular}
(a) Find the mean time taken
(b) Find the standard deviation for these times
(c) Find the median, upper and lower quartiles of these data.

\section*{AS/A Level Mathematics}

\section*{Box Plots and Outliers}

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

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- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 In a study of how students use their mobile telephones, the phone usage of a random sample of 11 students was examined for a particular week.

The total length of calls, y minutes, for the 11 students were
\[
17,23,35,36,51,53,54,55,60,77,110
\]
(a) Find the median and quartiles for these data.

A value that is greater than \(\mathrm{Q} 3+1.5 \times(\mathrm{Q} 3-\mathrm{Q} 1)\) or smaller than \(\mathrm{Q} 1-1.5 \times(\mathrm{Q} 3-\mathrm{Q} 1)\) is defined as an outlier.
(b) Show that 110 is the only outlier.
(c) Draw a box plot for these data indicating clearly the position of the outlier.


2 In a study of how much time students spend on social media, usage of a random sample of 15 students was examined for a particular day.

The total time of usage, \(x\) minutes, for the 15 students were
\[
6,25,39,62,65,74,80,94,125,127,154,159,184,210,251
\]
(a) Find the median and quartiles for these data.
(b) Show that there are no outliers.
(c) Draw a box plot for these data.


3 Children from two schools A and B took part in a competition to complete a puzzle. The distribution of times taken, to the nearest minute, by children in school A are shown in the box plot below.

(a) Write down the time by which \(75 \%\) of the children in school A had completed the puzzle.
(b) State what the two crosses ( \(\mathbf{x}\) ) represent on the box plot above. Interpret these in context.

For school B the shortest time taken was 6 minutes, the longest time taken was 22 minutes and the quartiles were 12,15 and 17 minutes respectively.
(c) Determine if there are any outliers.
(d) Draw a box plot for this information.

(e) Compare the two distributions.

\section*{AS/A Level Mathematics}

\section*{Histograms}

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

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- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 The partially completed histogram and the partially completed table show the times taken, to the nearest second, for a group of people to complete a puzzle.

\begin{tabular}{|c|c|}
\hline Time (seconds) & Frequency \\
\hline \(1-4\) & 2 \\
\hline \(5-8\) & 12 \\
\hline \(9-10\) & 8 \\
\hline \(11-15\) & 3 \\
\hline \(16-20\) & \\
\hline
\end{tabular}
(a) Complete the table.
(b) Complete the histogram

One of the participants is selected at random.
(c) Estimate the probability that this participant completed the puzzle in under 10 seconds.

2 A variable \(x\) was measured to the nearest whole number. 50 observations are given in the table below.
\begin{tabular}{|c|c|c|c|c|}
\hline\(x\) & \(5-8\) & \(9-13\) & \(14-20\) & \(21-24\) \\
\hline Frequency & 10 & 9 & 14 & 17 \\
\hline
\end{tabular}

A histogram was drawn and the bar representing the \(9-13\) class has a width of 2 cm and a height of 2.7 cm .

Find the width and height of the bar representing the \(14-20\) class.

3 A variable \(y\) was measured to the nearest whole number. 40 observations are given in the table below.
\begin{tabular}{|c|c|c|c|c|}
\hline\(y\) & \(10-19\) & \(20-24\) & \(25-28\) & \(29-30\) \\
\hline Frequency & 13 & 7 & 15 & 5 \\
\hline
\end{tabular}

A histogram was drawn and the bar representing the \(10-19\) class has a width of 2.5 cm and a height of 2.6 cm .
(a) Find the width and height of the bar representing the \(20-24\) class.
(b) Find the width and height of the bar representing the \(25-28\) class.

4 The distance travelled by 100 people to an event is summarised below.
\begin{tabular}{|c|c|}
\hline Distance (nearest mile) & Frequency \\
\hline \(0-9\) & 4 \\
\hline \(10-14\) & 19 \\
\hline \(15-18\) & 41 \\
\hline \(19-20\) & 26 \\
\hline \(21-25\) & 9 \\
\hline
\end{tabular}

A histogram was drawn and the bar representing the \(10-14\) class has a width of 3 cm and a height of 1.9 cm .
(a) Find the width and height of the bar representing the \(15-18\) class.
(b) Find the width and height of the bar representing the \(19-20\) class.

\section*{AS/A Level Mathematics}

\section*{Correlation and Regression}

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- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

\section*{Information}
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.

\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 Using the large data set Colin studied the relationship between rainfall ( \(r\) ) and temperature \((t)\) in Camborne. He took a sample of 12 days from May and June 1987 and obtained the following results.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Rainfall (cm) & 3.1 & 0.1 & 6 & 2.2 & 0.3 & 4.2 & 1.7 & 7.5 & 0.1 & 7.1 & 3.9 & 3.1 \\
\hline \begin{tabular}{c} 
Temperature \\
\(\left({ }^{\circ} \mathbf{C}\right)\)
\end{tabular} & 10.7 & 8.9 & 8.8 & 9.2 & 11.1 & 10.2 & 12.6 & 10.4 & 11.3 & 11.6 & 12.8 & 13.5 \\
\hline
\end{tabular}
(a) Use your knowledge of the large data set to explain why it is unlikely that Colin used a random sample.

Colin used a computer program to obtain the following statistics for Rainfall
\[
Q_{1}=1.35 \quad Q_{2}=3.1 \quad Q_{3}=4.65
\]
(b) Determine whether there are any outliers in the sample.
(c) Plot the information on a scatter graph
(d) Explain why a linear regression model may not be suitable for this data.

2 A football coach measured the heights and weights of 12 players, The data is shown below.
\begin{tabular}{|l|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Height (cm) & 188 & 194 & 178 & 175 & 185 & 175 & 188 & 193 & 180 & 190 & 181 & 169 \\
\hline Weight (kg) & 70 & 100 & 83 & 69 & 77 & 58 & 90 & 86 & 71 & 94 & 68 & 61 \\
\hline
\end{tabular}
(a) Draw a scatter graph for this information.
(b) Give an interpretation of the correlation between the height and weight of the footballers.

The equation of the regression line is \(w=1.37 h-173\)
(c) Give an interpretation of the gradient of this regression line.
(d) Use the equation of the regression line to estimate the weight of a player who is 170 cm tall.
(e) Comment on the reliability of your estimate in part (d), giving a reason for your answer.

3 Ross did an investigation into the relationship between study into temperature \((t)\) and total hours of sunshine \((s)\) at Heathrow. He finds that the equation of the regression line is \(s=0.25 t+1.3\).

Give an interpretation of the figure 0.25 figure in this regression line.

\section*{AS/A Level Mathematics}

\section*{Probability}

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- You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

\section*{Information}
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\section*{Advice}
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 The Venn diagram below shows three events \(A, B\) and \(C\).

(a) Write down two of the events that are mutually exclusive.

Events \(A\) and \(B\) are independent.
The probability of \(C\) is 0.3
(b) Find the values of \(p, q\) and \(r\).

2 The Venn diagram below shows three events \(A, B\) and \(C\).

(a) Write down two of the events that are mutually exclusive.

The probability of A is 0.4
The probability of A or B is 0.7
(b) Find the values of \(p, q\) and \(r\).
(c) State, giving a reason, whether of not the events A and B are statistically independent.

3 Raheem asks 50 people which sports they watch. The can chose from football, golf and hockey.

5 people watch all three sports.
8 people watch football and golf
7 people watch golf and hockey
9 people watch football and hockey
31 people watch football
13 people watch golf
17 people watch hockey.
(a) Draw a Venn diagram for this information.
(b) Two people are selected at random find the probability they both watch football.

4 For the events A and B.
The probability of A is 0.6
The probability of \(B\) is 0.5
The probability of neither A or B is 0.1 .
(a) Find \(\mathrm{P}(\mathrm{A}\) and B\()\)
(b) Draw a Venn diagram for this information.
(c) Determine whether A and B are independent.

5 Two events A and B are independent and \(\mathrm{P}(\mathrm{A})=0.4\) and \(\mathrm{P}(\mathrm{B})=0.3\)
(a) Find \(\mathrm{P}(\mathrm{A}\) and B\()\)
(b) Draw a Venn diagram for this information.

6 Two events A and B are mutually exclusive and \(\mathrm{P}(\mathrm{A})=0.4\) and \(\mathrm{P}(\mathrm{B})=0.3\)
(a) Write down \(\mathrm{P}(\mathrm{A}\) and B\()\)
(b) Draw a Venn diagram for this information.

7 Two events A and B are such that \(\mathrm{P}(\mathrm{A})=0.6\) and \(\mathrm{P}(\mathrm{B})=0.5\) and \(\mathrm{P}(\mathrm{A}\) and B\()=0.4\)
Draw a Venn diagram for this information.

1 A box contains 10 milk chocolates and 8 dark chocolates. Connor takes two chocolates at random. Find the probability Connor takes
(a) Two dark chocolates
(b) One milk chocolate and one dark chocolate.

2 A bag contains 10 blue counters, 8 red counters and 6 green counters.
Two counters are removed from the bag at random.
Find the probability that the two counters removed are:
(a) both red
(b) different colours

3 The probability a tennis player gets her first serve in court is \(65 \%\). If she gets her first serve in court the probability of winning the point is \(81 \%\).
The chance of getting her second serve in court is \(84 \%\) and if she gets he second serve in court the chance of winning the point is \(59 \%\).
If the tennis player fails to get her second serve in court she loses the point.
(a) Draw a tree diagram to show this information.
(b) Find the probability of the tennis player winning the point.

4 A company has three machines that produce a component. Machine A produces \(40 \%\) of the components. Machine B produces \(35 \%\) of the components and machine C produces \(25 \%\) of the components.

If a component is produced by machine A the chance that it will be faulty is \(3 \%\). If a component is produced by machine B the chance that it will be faulty is \(2 \%\). If a component is produced by machine C the chance that it will be faulty is \(1 \%\).
(a) Draw a tree diagram to show this information.

A component is selected at random. Find the probability:
(b) it is from machine A and faulty.
(c) it is faulty.

\title{
AS/A Level Mathematics
}

\section*{Discrete Random Variables}

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\section*{Advice}
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- Try to answer every question.
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1 A discrete random variable \(X\) has the probability function:
\begin{tabular}{|c|c|c|c|c|}
\hline\(x\) & 0 & 1 & 2 & 3 \\
\hline \(\mathrm{P}(X=x)\) & 0.2 & a & 0.3 & 0.25 \\
\hline
\end{tabular}
(a) Find the value of a
(b) Find \(\mathrm{P}(X>1.2)\)
(c) Construct a table for the cumulative distribution \(\mathrm{F}(x)\)

2 A random variable \(X\) has the probability function:
\[
\mathrm{P}(X=x)=\frac{(2 x-1)}{36} \quad x=1,2,3,4,5,6
\]
(a) Construct a table giving the probability function of \(X\).
(b) Find \(\mathrm{P}(1.4<X<3.9)\)
(c) Construct a table for the cumulative distribution \(\mathrm{F}(x)\)

3 A fair 6 sided die is rolled. The random variable \(Y\) represents the score on the die.
(a) Construct a table giving the probability function of \(Y\).
(b) Write down the name of this distribution

4 A discrete random variable \(X\) has the probability distribution:
\begin{tabular}{|c|c|c|c|c|}
\hline\(x\) & 0 & 1 & 2 & 3 \\
\hline \(\mathrm{P}(X=x)\) & 0.2 & \(a\) & 0.3 & \(b\) \\
\hline
\end{tabular}

Where \(a\) and \(b\) are constants.
The cumulative distribution \(\mathrm{F}(x)\) of \(X\) is given below.
\begin{tabular}{|c|c|c|c|c|}
\hline\(x\) & 0 & 1 & 2 & 3 \\
\hline \(\mathrm{~F}(x)\) & \(c\) & \(d\) & 0.78 & \(e\) \\
\hline
\end{tabular}

Where \(c, d\) and \(e\) are constants.
Find the values of \(a, b, c, d\) and \(e\).

\section*{AS/A Level Mathematics}

\section*{The Binomial Distribution and Hypothesis Testing}

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1 The discrete random variable \(X \sim \mathrm{~B}(15,0.35)\)
Find:
(a) \(\mathrm{P}(X=5)\)
(b) \(\mathrm{P}(X<4)\)
(c) \(\mathrm{P}(X \square 10)\)

2 The probability of Harry being late for school is 0.1 . Over a term of 30 days find the probability that Harry is late:
(a) Exactly one time
(b) More than four times
(c) Less than three times

3 The discrete random variable \(X \sim \mathrm{~B}(20,0.41)\)
(a) Find P ( \(3<X \square 7\) )

Previous research by a restaurant found that \(30 \%\) of customers will order a starter.
One one day a random sample of 40 customers is taken and 7 order a starter.
(b) Test at the 5\% significance level whether the proportion of customers ordering a starter has decreased.
(c) State the conclusion you would have reached if you tested at the \(10 \%\) significance level.

4 The discrete random variable \(X \sim \mathrm{~B}(30,0.58)\)
(a) Find \(\mathrm{P}(X \square 12)\)

A cafe expects \(30 \%\) of customers to order a coffee with their breakfast.
On one particular day a random sample of 40 customers that ordered breakfast is taken and 19 of them ordered a coffee.
(b) Test at the \(1 \%\) significance level whether the proportion of customers ordering a coffee had Increased. State your hypotheses clearly.
(c) State the conclusion you would have reached if a \(5 \%\) significance level had been used for this test. (1)

5 A company produces pens. The probability that any pen is defective is 0.08 .
(a) A sample of 15 pens is taken. Find the probability that 2 or more pens are defective.

An employee claims that the probability that a pen is defective is more than 0.08 . They take a sample of 20 pens and 3 are defective.
(b) Stating your hypothesis clearly, test the employee's claim at the 5\% significance level.

6 Andy plays tennis. The probability that Andy will get one of his serves in court is \(60 \%\).
Andy serves 20 times.
(a) Find the probability Andy gets:
(i) exactly 15 serves in court
(ii) more than 15 of the serves will be in court.
(b)Andy's coach thinks that the probability of Andy getting a serve in court has changed.

Andy serves 50 times in a set and 35 are in court.
Stating your hypothesis clearly, test the coach's claim at the \(10 \%\) significance level.
(Total for question 6 is 7 marks)

7 The probability of a bias dice landing on 6 is 0.4 . The dice is going to be rolled 20 times.
(a) Find the probability that the dice will land on 6 exactly 5 times.

Polly thinks that the probability that the dice will land on 6 is incorrect.
(b) Write down the hypotheses that should be used to test Polly's suspicion
(c) Using a 10\% significance level find the critical region for a two tailed test to test Polly's suspicion.

You should state the probability of rejection in each tail, which should be less than 0.05
(d) Find the actual significance level of a test based on your critical region from part (c)

Polly rolls the dice 20 times. The dice lands on six 11 times.
(e) Comment on Polly's suspicion in light of he experiment

\section*{Task 3 -}
\[
\begin{gathered}
\text { Mastering } \\
\text { Applied } \\
\text { Mechanics } \\
\text { Year } 1
\end{gathered}
\]

\section*{AS/A Level Mathematics}

\section*{Velocity-Time Graphs}

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\section*{Advice}
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1 A train accelerates from rest at station A to a velocity of \(32 \mathrm{~ms}^{-1}\). It maintains this speed for 72 seconds, until it decelerates uniformly to station B. The total journey time is 112 seconds and the magnitudes of the acceleration and deceleration are equal.
Find
(a) the time it takes the train to accelerate from rest to \(32 \mathrm{~ms}^{-1}\),
(b) sketch a velocity-time graph for the motion of the train between station \(A\) and station \(B\),
(c) calculate the distance between the two stations.

2 A train moves along a straight horizontal track between two stations, A and B. The train starts from rest and moves with acceleration of \(0.6 \mathrm{~ms}^{-2}\) for 40 seconds. The train moves at constant acceleration before decelerating at a rate of \(0.4 \mathrm{~ms}^{-2}\) until it reaches B.

The total distance between the two station is 4 km .
(a) sketch a velocity-time graph for the motion of the train between A and B,
(b) find the total time taken by the train to travel from A to B.
(Total for question 2 is 6 marks)
3 A particle, moving in a straight line with speed \(5 \mathrm{U} \mathrm{ms}^{-1}\), decelerates uniformly for 6 seconds which reduces its speed to \(2 \mathrm{U} \mathrm{ms}^{-1}\). It maintains this speed for a further 16 seconds before decelerating uniformly to rest in a further 2 seconds.
(a) sketch a velocity-time graph for this information,
(b) find an expression for each of the decelerations in terms of U .

Given the total distance is 220 m
(c) find the value of U .

4 A car starts from rest and accelerates at a constant rate to the speed of \(\mathrm{V} \mathrm{ms}^{-1}\) in 6 seconds. The car maintains this speed for 50 seconds before decelerating to rest.
The magnitude of the deceleration is 1.5 times this magnitude of the acceleration.
(a) show the total time taken for the journey is 60 seconds,
(b) sketch a velocity-time graph for this information,

Given the total distance is 1320 m
(c) find the value of V .

\section*{AS/A Level Mathematics}

\section*{SUVAT}

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\section*{Advice}
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1 A particle moves from A to \(B\) under constant acceleration.
The distance between A and B is s .
The speed at \(A\) is \(u\). The speed at \(B\) is \(v\).
The time taken is t .
The acceleration is a.
\begin{tabular}{lllll} 
(a) Given & \(\mathrm{s}=50 \mathrm{~m}\) & \(\mathrm{u}=0 \mathrm{~ms}^{-1}\) & \(\mathrm{v}=20 \mathrm{~ms}^{-1}\) & Find a and t. \\
(b) Given & \(\mathrm{s}=200 \mathrm{~m}\) & \(\mathrm{a}=2 \mathrm{~ms}^{-2}\) & \(\mathrm{v}=30 \mathrm{~ms}^{-1}\) & Find t and u. \\
(c) Given & \(\mathrm{s}=85 \mathrm{~m}\) & \(\mathrm{t}=5 \mathrm{~s}\) & \(\mathrm{v}=20 \mathrm{~ms}^{-1}\) & Find a and u. \\
(d) Given & \(\mathrm{s}=100 \mathrm{~m}\) & \(\mathrm{a}=2 \mathrm{~ms}^{-2}\) & \(\mathrm{t}=4 \mathrm{~s}\) & Find u and v. \\
(e) Given & \(\mathrm{v}=10 \mathrm{~ms}^{-1}\) & \(\mathrm{a}=1.5 \mathrm{~ms}^{-2}\) & \(\mathrm{t}=3 \mathrm{~s}\) & Find s and u.
\end{tabular}
(c) Given \(\mathrm{s}=85 \mathrm{~m} \quad \mathrm{t}=5 \mathrm{~s} \quad \mathrm{v}=20 \mathrm{~ms}^{-1} \quad\) Find a and u .
(d) Given \(\mathrm{s}=100 \mathrm{~m} \quad \mathrm{a}=2 \mathrm{~ms}^{-2} \quad \mathrm{t}=4 \mathrm{~s} \quad\) Find u and v .
(e) Given \(\quad v=10 \mathrm{~ms}^{-1} \quad \mathrm{a}=1.5 \mathrm{~ms}^{-2} \quad \mathrm{t}=3 \mathrm{~s} \quad\) Find s and u .
(Total for question 1 is 10 marks)
2 A ball is projected vertically upwards with a speed of \(20 \mathrm{~m} \mathrm{~s}^{-1}\) from a point \(h\) metres above the ground. The ball hits the ground 5 s later. Find
(a) the value of \(h\),
(b) the speed of the ball as it hits the ground.
(Total for question 2 is 6 marks)
3 A car passes point A with a speed of \(20 \mathrm{~km} / \mathrm{h}\). The car accelerates at a constant rate and 10 seconds later it passes point B with a speed of \(70 \mathrm{~km} / \mathrm{h}\). Find
(a) the acceleration of the car in \(\mathrm{ms}^{-1}\),
(b) the distance AB .

4 A stone is dropped from a point 120 m from the ground. Find
(a) the time it takes for the stone to reach the ground,
(b) the speed at which the stone hits the ground.

5 A particle moves along a straight line, from point X to point Y , with constant acceleration.
The distance XY is 120 m . The particle takes 8 seconds to move from X to Y and the speed of the particle at Y is double the speed of the particle at X .
Find
(a) the speed of the particle at X ,
(b) the acceleration of the particle.

6 A car passes point A with a speed of \(5 \mathrm{~ms}^{-1}\). The car accelerates at a constant rate and 8 seconds later it passes point \(B\) with a speed of \(20 \mathrm{~ms}^{-1}\). Find
(a) the acceleration of the car,
(b) the distance AB ,
(c) the time it takes the car to reach the midpoint of AB .

7 A train, moving with constant acceleration, passes through three points \(\mathrm{A}, \mathrm{B}\) and C , where \(\mathrm{AB}=40 \mathrm{~m}\) and \(B C=60 \mathrm{~m}\). The train passes through A with a speed of \(10 \mathrm{~ms}^{-1}\) and 6 seconds later passes through \(C\). Find
(a) the acceleration of the train,
(b) the speed at which the train passes though B.
(c) The time it take for the train to move between B and C.

8 A stone is projected vertically upwards with a speed \(18 \mathrm{~ms}^{-1}\) from a point 2 m above the ground. Find
(a) the greatest height reached by the stone,
(b) the speed at which the stone hits the ground.
(c) the time between the instant the stone is projected and when it hits the ground.
(Total for question 8 is 6 marks)
9 A car passes three posts \(\mathrm{P}, \mathrm{Q}\) and R , on a straight horizontal road. The distance \(\mathrm{PQ}=50 \mathrm{~m}\). The distance \(Q R=100 \mathrm{~m}\). The car, moving with constant acceleration, takes 2 seconds to travel from \(P\) to \(Q\) and 3 seconds to travel from Q to R .
(a) the acceleration of the car,
(b) the speed car at the instant it passes Q .

10 A particle is projected vertically upwards from a point 1.5 m above the ground with a speed of \(10 \mathrm{~ms}^{-1}\). Find
(a) the greatest height reached by the particle,
(b) the time for which the particle is more than 3 m above the ground.

\section*{AS/A Level Mathematics}

\section*{2D Vectors}

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\section*{Advice}
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1 Three forces \((3 \mathbf{i}+2 p \mathbf{j}) \mathrm{N},(-2 \mathbf{i}+5 \mathbf{j}) \mathrm{N}\) and \((4 q \mathbf{i}+5 \mathbf{j}) \mathrm{N}\) act on a particle \(A\).
Given \(A\) is in equilibrium find the values of \(p\) and \(q\).

2 A particle \(P\) of mass 0.8 kg moves under the action of a force FN .
The acceleration of P is \((-3 \mathbf{i}+5 \mathbf{j})\).
(a) Find the angle between the acceleration and the vector \(\mathbf{i}\).
(b) Find the magnitude of F .

3 A particle \(P\) of mass 2 kg moves with constant acceleration under the action of a force F N . The initial velocity of \(P\) is \((-2 \mathbf{i}+6 \mathbf{j}) \mathrm{ms}^{-1}\) and after 4 seconds the velocity of \(P\) is \((7 \mathbf{i}+2 \mathbf{j}) \mathrm{ms}^{-1}\).

Find, to three significant figures:
(a) the magnitude of the acceleration,
(b) the angle between F and the vector \(\mathbf{i}\).

4 The resultant of two forces \(\mathrm{F}_{1}\) and \(\mathrm{F}_{2}\) is \((\mathbf{i}-14 \mathbf{j}) \mathrm{N}\).
Given that \(\mathrm{F}_{1}=(2 p \mathbf{i}-4 q \mathbf{j}) \mathrm{N}\) and \(\mathrm{F}_{2}=(3 q \mathbf{i}+4 p \mathbf{j}) \mathrm{N}\) find the values of \(p\) and \(q\).

5 A particle \(P\) moves with a constant velocity of \((4 \mathbf{i}-\mathbf{j}) \mathrm{ms}^{-1}\).
(a) Find the speed of \(P\).
(b) Find the direction of motion of \(P\), giving your answer as a bearing.

6 A particle \(P\) moves with constant acceleration \((3 \mathrm{i}-4 \mathrm{j}) \mathrm{ms}^{-2}\).
At time \(t=0\), P has speed \(u \mathrm{~ms}^{-1}\)
At time \(t=3\), P has velocity \((-5 \mathrm{i}+2 \mathrm{j}) \mathrm{ms}^{-1}\)
Find the value of \(u\).

7 The resultant of two forces \(\mathrm{F}_{1}\) and F is parallel to \(\mathbf{i}+\mathbf{j}\).
Given that \(\mathrm{F}_{1}=(3 \mathbf{i}-2 \mathbf{j}) \mathrm{N}\) and \(\mathrm{F}_{2}=(p \mathbf{i}+2 p \mathbf{j}) \mathrm{N}\), where p is a positive constant.
Find the value of \(p\).

\section*{AS/A Level Mathematics}

\section*{\(\mathrm{F}=\mathrm{ma}\)}

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1 Two particles \(A\) and \(B\) have mass of 2 kg and \(m \mathrm{~kg}\) respectively, where \(m<2\). The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially \(A\) is 3 m above horizontal ground.
The string is released from rest with the string taut and the hanging parts of the string vertical.
After \(A\) has been descending for 2.5 s it strikes the ground.
Particle \(A\) reaches the ground before \(B\) reaches the pulley.

(a) Show that the acceleration of \(A\) as it descends is \(0.96 \mathrm{~ms}^{-2}\)
(b) Show that the mass of \(B\) is 1.64 kg
(c) State how you have used the information that the string is inextensible.

2 A particle \(A\) of mass 5 kg rests on a smooth horizontal table. Particle \(A\) is attached to one end of a light inextensible string which passes over a smooth pulley fixed to the edge of the table. The other end of the string is attached to particle \(B\) of mass 4 kg which hangs freely below the pulley 1.4 m above the ground. The system is released from rest with the string taut. Particle \(A\) does not reach the pulley before \(B\) reaches the ground.

(a) Find the tension in the string before \(B\) hits the ground.
(b) Find the time taken by \(B\) to reach the ground.
(Total for question 2 is 9 marks)
3 A car of mass 750 kg pulls a trailer of mass 300 kg along a straight horizontal road using a light towbar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 250 N and 100 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1600 N . Find
(a) the acceleration of the car and trailer,
(b) the magnitude of the tension in the towbar.

The car is moving along the road when the driver sees a hazard ahead. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F newtons and the car and trailer decelerate. Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 80 N ,
(c) find the value of F .

4 A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope.
The mass of the car is 1400 kg . The mass of the trailer is 700 kg . The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively.
The driving force on the car, due to its engine, is 2380 N. Find
(a) the acceleration of the car,
(b) the tension in the tow-rope
(c) state how you have used the assumption that the car and trailer are modelled as particles.

When the car and trailer are moving at \(12 \mathrm{~m} \mathrm{~s}^{-1}\), the tow-rope breaks. Assuming that the driving force on the car and the resistances to motion are unchanged,
(d) find the distance moved by the car in the first 4 s after the tow-rope breaks.
(Total for question \(\mathbf{4}\) is 13 marks)
5


Two particles \(P\) and \(Q\), of mass 4 kg and 6 kg respectively, are joined by a light horizontal rod.
The particles are initially at rest when a constant force F of magnitude 30 N is applied to \(Q\), as shown in the diagram. The force is applied for 5 s .
During the motion, the resistance to \(P\) has a constant magnitude of 2 N and the resistance to \(Q\) has a constant magnitude of 4 N . Find
(a) the acceleration of the particles as the system moves under the action of the 30 N force.
(b) the speed of the particles after 5 seconds.
(c) the tension in the string as the system moves under the action of the 30 N force.

After 5 seconds the force is removed and the system decelerates to rest.
The resistances to motion are unchanged.
(d) Find the distance moved by \(P\) as the system decelerates.
(e) Find the thrust in the rod as the system decelerates.

\section*{AS/A Level Mathematics}

\section*{Variable Acceleration}

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\section*{Advice}
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1 The acceleration of a particle after \(t\) seconds is given by \((4 t-8) \mathrm{ms}^{-2}\).
Given the velocity \((v)\) of the particle is \(6 \mathrm{~ms}^{-1}\) when \(t=0\).
(a) Find \(v\) in terms of \(t\).
(b) Find the distance between the two points the particle is instantaneously at rest.

2 The velocity of a particle after t seconds is given by \(v=(6 t-2) \mathrm{m} \mathrm{s}^{-1}\).
After 5 seconds the displacement is 75 m from O .
(a) Find an expression for the displacement.
(b) Find the displacement after 10 seconds.

3 A particle \(P\) moves in a straight line so that, at time \(t\) seconds, its acceleration \(a \mathrm{~m} \mathrm{~s}^{-2}\) is given by

At \(t=0, P\) is at rest. Find the speed of \(\left\{\begin{array}{ll}\frac{27}{4 t-t^{2}} & t>3 \\ \text { when } & 0 \leqslant t \leqslant 3\end{array}\right\}\)
(a) \(t=3\)
(b) \(t=6\)

4 A particle \(P\) moves in a straight line so that, at time \(t\) seconds, its velocity \(v \mathrm{~m} \mathrm{~s}^{-1}\) is given by
\[
v=\left\{\begin{array}{lc}
7 \mathrm{t}-t^{2} & 0 \preccurlyeq t \preccurlyeq 5  \tag{3}\\
10-2 \mathrm{t} & t>5
\end{array}\right\}
\]
(a) Find the acceleration of P when \(\mathrm{t}=4\)
(b) Find the total distance traveled by P in the first 10 seconds.

5 The velocity of a particle after t seconds is given by \(v=\left(6 t-2 t^{2}\right) \mathrm{m} \mathrm{s}^{-1}\).
Find the time at which the acceleration of the particle is zero.
(Total for question 5 is \(\mathbf{3}\) marks)
6 A particle \(P\) moves on the \(x\) axis. The acceleration of \(P\) at time \(t\) seconds is \((6 t-24) \mathrm{m} \mathrm{s}^{-2}\) measured in the positive \(x\) direction. Initially the particle is at \(O\) with a velocity of \(60 \mathrm{~m} \mathrm{~s}^{-1}\).
(a) Show that the particle will never travel in the negative \(x\) direction.
(b) Find the distance travelled by the particle in the first 10 seconds.

7 A particle \(P\) moves in a straight line such that at \(t\) seconds, \(t \geq 0\), its velocity, \(v \mathrm{~ms}^{-1}\) is given by:
\[
\begin{equation*}
v=12-2 t^{2} \tag{3}
\end{equation*}
\]

Find:
(a) the distance travelled by \(P\) in the first second,
(b) the value of \(t\) when \(P\) changes direction of motion,
(c) the value of \(t\) at the instant \(P\) returns to its starting point.

8 A particle \(P\) travels along a straight line through a point \(O\) so that at time \(t \mathrm{~s}\) after passing through \(O\) its displacement from \(O\) is \(x \mathrm{~m}\), where \(x=t^{3}-15 t^{2}+62 t\)

Find
(a) the initial velocity of \(P\),
(b) the value of t for which \(P\) has zero acceleration.

9 A particle \(P\) travels along a straight line through a point \(O\) so that at time \(t \mathrm{~s}\) after passing through \(O\) its displacement from \(O\) is \(x \mathrm{~m}\), where \(x=2 t^{3}-18 t^{2}+48 t\)

Find
(a) the times when P is instantaneously at rest,
(b) the total distance travelled in the first 5 seconds.

10 A particle \(P\) moves on the \(x\) axis.
The acceleration of P at time \(t\) seconds, \(t \geq 0\), is \((3 t+5) \mathrm{m} \mathrm{s}^{-2}\) in the positive \(x\) direction.
When \(t=0\), the velocity of \(P\) is \(2 \mathrm{~m} \mathrm{~s}^{-1}\) in the positive \(x\) direction.
When \(t=\mathrm{T}\), the velocity of P is \(6 \mathrm{~m} \mathrm{~s}^{-1}\) in the positive \(x\) direction.
Find the value of T .

11 The displacement of a particle \(P\) from the origin after \(t\) seconds is given by \(\mathrm{s}=t^{2}(t+k) \mathrm{m}\)
Given P is at instantaneous rest when \(t=4\).
Find the acceleration of \(P\) when \(t=10\).
(Total for question 11 is \(\mathbf{8}\) marks)
12 The velocity of a particle after t seconds is given by \(v=\left(6 t-2 t^{2}\right) \mathrm{m} \mathrm{s}^{-1}\).
When \(t=0\) the particle is at the origin \(O\).
Find an the distance of the particle from \(O\) when the particle comes to instantaneous rest.```

